THE SHIN-CHU-MON mathematics for 7th grade
Excellent collection of the newest questions about mathematics for 7th grade with detailed explanations
Mathematics
7th grade

Special features of this book

This book mainly focuses on common and typical problems and classifies them carefully to help you efficiently learn how to solve various types of problems.

Problems are arranged in order from easy to difficult so that you can smoothly acquire both basic and applied skills.

Structure of this book

Key points of study
Important points are summarized at the beginning of each chapter, which is divided into several sections.

Let's learn the basics / Questions
First, you learn how to solve typical problems about each key point. Next, you try to solve similar problems by yourself to make sure that you understand what you have learned.

Exercises
You can check your understanding at the end of each section.

Comprehension test / End-of-chapter problems
The former contains mid-level questions, and the latter contains higher-level ones.

Let's review
There are problems related to each chapter so that you can easily review all you have learned in the first year of junior high school.

Final test
You can take a test on a 100-point scale to check how much you understood.

Complements
This optional section deals with what is beyond the curriculum guidelines. You can study it according to your interests.

Note: In this book, L represents “liter”; km/h, m/min and m/s represent “kilometers per hour,” “meters per minute” and “meters per second,” respectively.
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Let's review what you have learned at elementary school

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3 Multiplication and division
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Complements
How to solve inequalities
Let's review what you have learned at elementary school

1 Numbers and calculations

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<th>6 Permutations</th>
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</table>

1 Calculations with integers

- Calculate.
  - (1) \(15 - 9 + 12\)
  - (2) \(21 \times 2 \div 6\)
  - (3) \(15 + 30 \div 5\)
  - (4) \(17 + 2 \times (18 - 12)\)
  - (5) \(15 + 30 \div 5\)
  - (6) \(20 \div 5\)
  - (7) \(4 \times 8 - 54 \div 6\)
  - (8) \(40 - 2 \times (6 + 20 \div 5)\)

2 Calculations with decimals

- Calculate.
  - (1) \(1.56 - 0.89\)
  - (2) \(2.8 - 3.5 - 4.7\)
  - (3) \(0.42 \times 8\)
  - (4) \(1.6 \times 0.7\)
  - (5) \(0.72 \div 9\)
  - (6) \(5.2 \div 0.4\)
  - (7) \(6 \times (5.4 - 3.9)\)
  - (8) \(4 \times 0.52 - 0.32 \div 8\)

3 Calculations with fractions

- Calculate.
  - (1) \(\frac{3}{8} + \frac{1}{4}\)
  - (2) \(\frac{1}{6} - \frac{2}{3}\)
  - (3) \(\frac{3}{4} + \frac{1}{3} - \frac{5}{6}\)
  - (4) \(\frac{5}{12} \times \frac{4}{15}\)
  - (5) \(\frac{9}{10} \div \frac{3}{5}\)
  - (6) \(\frac{2}{9} \times \frac{3}{5} \div \frac{4}{15}\)
  - (7) \(\frac{1}{2} \div \frac{1}{3} \times \frac{1}{4}\)
  - (8) \(\frac{3}{4} \div \left(\frac{2}{3} \div \frac{1}{2}\right)\)
  - (9) \(\frac{3}{5} \div \frac{2}{3} - \frac{3}{4} \times \frac{2}{5}\)
Let's review what you have learned at elementary school.

4 Multiples, Factors  Answer the following questions.

(1) Write four multiples of 7 starting with the smallest one, excluding 0.

(2) How many multiples of 6 are there in integers from 1 to 100?

(3) Find all the factors of the following numbers.
   1. 8
   2. 13
   3. 20

(4) Find the greatest common factor and the least common multiple of the following numbers.
   1. (6, 9)
   2. (15, 30)
   3. (8, 20, 24)

(5) Find the smallest double-figure integer that leaves a remainder of 4 both when divided by 9 and when divided by 12.

(6) When you divide 38 and 54 by an integer, the remainders are both 6. Find all integers that satisfy the condition.

5 Word problems  Answer the following questions.

(1) There is a school that has 420 students, \( \frac{11}{20} \) of whom are boys. How many boys and girls are there respectively in this school?

(2) There is a rectangular sheet of paper 56 cm long and 96 cm wide. In order to cut it into as large same-sized squares as possible without odd pieces, how many centimeters should the length of a side of the square be?

(3) Arrange equilateral triangles with sides of 1 cm in a row to make the figure below.

\[ \text{How many centimeters is the circumference of the figure made of 20 triangles?} \]

6 Permutations  Answer the following questions.

(1) When you make three-digit integers by arranging 3 cards 1, 2, and 3, how many different numbers can you make?

(2) Four teams, A, B, C, and D are going to play soccer games. In a round-robin format, where each team meets every other team once, how many games will be played?

(3) When A and B play the game of "rock-paper-scissors," how many combinations of rock, paper, and scissors are there?
Let's review what you have learned at elementary school

## 2 Units, Average, Speed

### Contents
1. Units
2. Numbers or amounts per unit
3. Average
4. Speed

### 1 Units
Express the quantities using the unit shown in [ ].

- (1) 3 km [m]
- (2) 1.8 m [cm]
- (3) 1 m² [cm²]
- (4) 1700 g [kg]
- (5) 0.9 g [mg]
- (6) 2 minutes 30 seconds [minute]

### 2 Numbers or amounts per unit
Answer the following questions.

- (1) A car used 20 L of gasoline to run 170 km. How many kilometers did it run per liter?

- (2) A city's area is 86 km² and its population is 120000. Calculate the population density of this city and round the value to obtain the leading two digits.

### 3 Average
Answer the following questions.

- (1) Calculate the average of the following quantities.
  - 1) 80 points, 68 points, 71 points
  - 2) 42 g, 51 g, 48 g, 43 g

- (2) Satoko's average score on Japanese, science and social studies tests was 78. If her score on math is 86 points, how many points is her average score of the four subjects?

### 4 Speed
Answer the following questions.

- (1) A train runs 296 km in 4 hours. Find its speed per hour.

- (2) When a car runs 42 kilometers per hour, how many kilometers does it travel in 40 minutes?

- (3) How many hours and how many minutes does it take to walk 6.3 km at a speed of 60 meters per minute?

- (4) Realizing that Takashi had left something at home, his father got on his bicycle and left the house to catch up with Takashi, who was already 600 m away. If the father rode at 200 m per minute and Takashi walked at 50 m per minute, how many minutes after the father left the house did he catch up with Takashi?
Let's review what you have learned at elementary school.

### 3 Ratios, Graphs, Proportions and inverse proportions

**Contents**
1. Percentages and *Buai*  
2. Relative amounts  
3. Proportions and graphs  
4. Ratios  
5. Proportions and inverse proportions

### 1 Percentages and *Buai*
Change the following decimal ratios into percentage and *buai*.

<p>| | | | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.2</td>
<td>(2)</td>
<td>0.62</td>
<td>(3)</td>
</tr>
</tbody>
</table>

### 2 Relative amounts
Fill in [ ] with a suitable number.

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<tr>
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</thead>
<tbody>
<tr>
<td>(1)</td>
<td>60 yen is [ ] <em>wari</em> of 200 yen.</td>
<td>(2)</td>
<td>42 meters is [ ] % of 600 meters.</td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>60% of 300 m² is [ ] m².</td>
<td>(4)</td>
<td>5 <em>wari</em> 4 <em>bu</em> of 600 g is [ ] g.</td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>The payment for an 800-yen article at a 40% discount is [ ] yen.</td>
<td></td>
<td></td>
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</table>

### 3 Proportions and graphs
The circular graph on the right shows the land areas used for growing different fruits in an orchard. Answer the following questions.

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<tbody>
<tr>
<td>(1)</td>
<td>What percentages of land areas are used for strawberries, grapes and pears, respectively?</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(2)</td>
<td>If the whole area of the orchard is 7200 m², how many m² of land areas are used for strawberries, grapes and pears, respectively?</td>
<td></td>
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</table>

### 4 Ratios
Answer the following questions.

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<tbody>
<tr>
<td>(1)</td>
<td>Simplify the following ratios.</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(2)</td>
<td>Fill in [ ] with a suitable number.</td>
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<p>| | | | | |</p>
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<tbody>
<tr>
<td>(1)</td>
<td>8 : 24</td>
<td>(2)</td>
<td>35 : 49</td>
<td>(3)</td>
</tr>
<tr>
<td>(4)</td>
<td>$\frac{4}{3} : $ $\frac{2}{5}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | |</p>
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<tbody>
<tr>
<td>(1)</td>
<td>2 : 5 = 6 : [ ]</td>
<td>(2)</td>
<td>[ ] : 12 = 7 : 4</td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>[ ] : 1.6 = 5 : 4</td>
<td></td>
<td></td>
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</tbody>
</table>

### 5 Proportions and inverse proportions
*y* is proportional to *x* in the table (1), while *y* is inversely proportional to *x* in the table (2). Fill in the blanks ① to ④.

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</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>Proportion</td>
<td>(2)</td>
<td>Inverse proportion</td>
<td></td>
</tr>
<tr>
<td>[x \quad 2 \quad 4 \quad 8 \quad 3]</td>
<td>[y \quad 8 \quad \boxed{1} \quad 24 \quad \boxed{3} \quad 40]</td>
<td>[x \quad 2 \quad 3 \quad \boxed{2} \quad 6 \quad \boxed{4}]</td>
<td>[y \quad 18 \quad 1 \quad \boxed{9} \quad \boxed{3} \quad 4]</td>
<td></td>
</tr>
</tbody>
</table>

*1 *wari* is $\frac{1}{10}$ or 10%, 1 *bu* is $\frac{1}{100}$ or 1%, 1 *rin* is $\frac{1}{1000}$ or 0.1%. These expressions are used in Japan and are called *buai*.  

*7*
Let's review what you have learned at elementary school.

Figures and Measurements

1. **Areas**
   - Calculate the area of the figures (1) to (5) and that of the shaded part of the figure (6). Use 3.14 for the ratio of the circumference of a circle to its diameter.
   - (1) Rectangle
   - (2) Triangle
   - (3) Trapezoid
   - (4) Rhombus
   - (5) Circle
   - (6) Shaded part of a figure

2. **Circumferences**
   - Calculate the circumference and area of the shaded part of the following figures. Use 3.14 for the ratio of the circumference of a circle to its diameter.
   - (1) Circle
   - (2) Elliptical figure

3. **Angles**
   - Find the angle $x$ of the following figures.
   - (1) Line $l$ is parallel to line $m$
   - (2) Isosceles triangle
   - (3) Quadrilateral
   - (4) Overlapping triangular rulers
Let's review what you have learned at elementary school.

4 Congruent figures  In the parallelogram ABCD, shown on the right, E is the intersection of the two diagonals. Answer the following questions.

\(\square\) 1) In the triangle that is congruent to triangle ABC, which side corresponds to side AB?

\(\square\) 2) In the triangle that is congruent to triangle BCE, which angle corresponds to angle (iii)?

5 Enlargement and reduction  Triangle DEF shown on the right is an enlargement of triangle ABC. Answer the following questions.

\(\square\) 1) How many centimeters is side DF?

\(\square\) 2) How many degrees is angle x?

6 Symmetric figures  Choose all the figures having line symmetry or point symmetry among those below and answer using the symbols (i) to (iv).

(i) Triangle  (ii) Parallelogram  (iii) Rectangle  (iv) Rhombus

7 Volumes  Calculate the volume of the solids below. Use 3.14 for the ratio of the circumference of a circle to its diameter.

\(\square\) 1) (Cuboid)

\(\square\) 2) (Triangular prism)

\(\square\) 3) (Cylinder)

8 Three-dimensional positions  The figure shown on the right is a net of a cube.

\(\square\) 1) Which side corresponds to side AB?

\(\square\) 2) Which face is parallel to face (v)?

\(\square\) 3) How many sides are perpendicular to one face in a cube?
1 Positive numbers. Negative numbers

(1) Numbers that are greater than 0 are called positive numbers. They are sometimes shown with a positive sign (+).
(2) Numbers that are smaller than 0 are called negative numbers. They are always shown with a negative sign (−).
(3) There are positive integers and negative ones. Positive integers are called natural numbers.
(4) Positive and negative numbers are used to express quantities that have opposite properties.

2 The number line. Absolute values. Size of numbers

(1) On the number line, the point that shows 0 is called the origin. Starting at the origin, the right direction is called the positive direction, and the left direction is the negative direction.
(2) On the number line, the value that shows the distance from the origin to a point representing a number is called its absolute value.
(3) The following can be said about the size of signed numbers.
   ① Positive numbers are greater than 0, and negative numbers are smaller than 0.
      (negative numbers) < 0 < (positive numbers)
   ② Positive numbers become greater as their absolute values increase.
   ③ Negative numbers become smaller as their absolute values increase.

3 Addition with positive and negative numbers

(1) The sum of two numbers with the same sign is that of their absolute values with the common sign.
(2) The sum of two numbers with different signs is the difference of their absolute values with the sign of the number whose absolute value is greater than the other. The sum of two numbers with the same absolute value and different signs is 0.

4 Subtraction with positive and negative numbers

In order to subtract a positive or negative number, add the same number with the different sign.

5 Multiplication with positive and negative numbers

(1) The product of two numbers with the same sign is that of their absolute values with the positive sign.
(2) The product of two numbers with different signs is that of their absolute values with the negative sign.
(3) The product of a number multiplied by itself is called powers, and the small number to the upper right is called an exponent. The exponent indicates how many times the number is multiplied.

6 Division with positive and negative numbers

(1) The quotient of two numbers with the same sign is that of their absolute values with the positive sign.
(2) The quotient of two numbers with different signs is that of their absolute values with the negative sign.
(3) Dividing by a positive or negative number is the same as multiplying by its reciprocal.

7 Laws of calculations

(1) the commutative law of addition \( a + b = b + a \)
(2) the associative law of addition \( (a + b) + c = a + (b + c) \)
(3) the commutative law of multiplication \( a \times b = b \times a \)
(4) the associative law of multiplication \( (a \times b) \times c = a \times (b \times c) \)
(5) the distributive law \( a \times (b + c) = a \times b + a \times c, (a + b) \times c = a \times c + b \times c \)
(6) The sign of the product of several numbers is negative (−) if they include an odd number of negative signs, and it is positive (+) if they include an even number of them. The absolute value of the product is equal to the product of the absolute values of the component numbers.
Signed numbers

Let's learn the basics

1. How to express negative numbers (numbers smaller than 0)
   For example, the number that is 7 smaller than 0 is written as \(-7\) and read as negative 7.

2. How to express positive numbers (numbers greater than 0)
   For example, the number that is 5 greater than 0 is written as \(+5\) and read as positive 5.

3. How to express quantities that have opposite properties
   For example, when an income of 100 yen is expressed as \(+100\) yen, the expense of 200 yen is \(-200\) yen.

1. Express the following numbers using a positive or negative sign.
   - (1) the number 9 smaller than 0
   - (2) the number 6 greater than 0
   - (3) the number \(\frac{1}{4}\) smaller than 0
   - (4) the number 8 greater than 0
   - (5) the number 3 smaller than 0
   - (6) the number \(\frac{2}{3}\) greater than 0

2. Answer the following questions.
   - (1) When “two years from now” is expressed as +2 years, how do you say “three years ago”?
   - (2) When a profit of 300 yen is expressed as +300 yen, how do you say a loss of 120 yen?
   - (3) When going 5 steps up the stairs is expressed as +5 steps, how do you say going 6 steps down?

3. Write the following phrases using a positive number.
   - (1) -5 meters high
   - (2) -10 minutes fast
   - (3) an increase of -6 kilograms

4. Write the following phrases using a negative number.
   - (1) +7 kilogram heavy
   - (2) an expense of +500 yen
   - (3) a decrease of 150 people

5. List all the numbers in \[ \{-8, +7, -2.3, 0, -\frac{1}{7}, 5, +\frac{1}{8}, +0.5\} \] that apply to (1) to (3).
   - (1) Negative numbers
   - (2) Negative integers
   - (3) Natural numbers

Points

1. Numbers like +2 or +4.5 are positive numbers, and numbers like -2 or \(-\frac{1}{3}\) are negative numbers.
2. Integers include positive integers, 0, negative integers. Positive integers are called natural numbers, too.

Exercises ← P14
Let's learn the basics.  

Signs like $>$ or $<$ are called inequality signs. They are used to show the relative size between two numbers.

1. The relative size of 8 to 4 is expressed as $8 > 4$ with an inequality sign.
2. The relative size between the three numbers 2, 4, and 8 is expressed as $2 < 4 < 8$ or $8 > 4 > 2$.

1. The relative size between $-3$ and $+2$  
   Numbers on the number line become greater as they go farther to the right. So $+2$ is greater than $-3$. This relation is expressed as $-3 < +2$ with an inequality sign.

2. The relative size between $-2$ and $-5$  
   $-2$ is greater than $-5$ because $-2$ is to the right of $-5$. This relation is expressed as $-5 < -2$.

6. On the number line below, what are the numbers represented by points A, B, C, and D?

   ![Number line diagram]

7. Mark the points representing the following numbers on the number line below.
   - (1) $+7$
   - (2) $-3$
   - (3) $-6.5$

   ![Number line diagram]

8. Express the relative size between the numbers below with an inequality sign.
   - (1) $+3, -5$
   - (2) $-4, 0$
   - (3) $-6, -1$
   - (4) $-8, +7$
   - (5) $-9, -10$
   - (6) $0, -13$
   - (7) $-0.7, -2$
   - (8) $-1.9, +0.7, -2.6$
   - (9) $-\frac{5}{3}, +\frac{1}{2}, -\frac{1}{4}$

9. Choose the greatest number and the smallest number among those in each group.
   - (1) $-15, -21, +18, -37, +9$
   - (2) $\frac{4}{5}, -\frac{2}{5}, -\frac{5}{2}, +\frac{1}{2}, -\frac{3}{5}, -\frac{6}{5}$

Points

1. The number line is made as follows: On a line marked a point indicating 0, from which points are marked at even intervals. The points to the right represent positive numbers $+1, +2, +3$, etc., and those to the left represent negative numbers $-1, -2, -3$, etc.
2. The point indicating 0 on the number line is called the origin. To the right of the origin is the positive direction, and to the left of the origin is the negative direction.
Let's learn the basics

3 Absolute values

1. Positive numbers and absolute values
   The distance from the origin to the point representing +3 on the number line is 3, so the absolute value of +3 is 3.

2. Negative numbers and absolute values
   The distance from the origin to the point representing −4 on the number line is 4, so the absolute value of −4 is 4.

3. Numbers without a sign
   Numbers without a sign are positive numbers. The distance from the origin to the point representing 2 is 2, so the absolute value of 2 is 2.

10 Find the absolute value of the following numbers.

1) +9
2) −6
3) −\(\frac{2}{3}\)
4) 0.7
5) −3.3

11 List all the numbers that apply to the following.

1) the number whose absolute value is 8
2) the integer whose absolute value is smaller than 4
3) the integer whose absolute value is larger than 2 and smaller than 6
4) the integer whose absolute value is 10 or more and 12 or less

12 Arrange the numbers in each group in ascending order of absolute value.

1) +18, −13, −6
2) −26, 0, 10
3) +5, −32, +20
4) −61, +57, 48
5) −7.3, −5.2, +6.5, −4.9
6) \(\frac{3}{4}, \frac{-5}{8}, \frac{-5}{6}, \frac{-2}{3}\)
7) 0.2, −1.2, −\(\frac{2}{5}\), \(\frac{3}{4}\)
8) 3, −3.1, 2.9, \(\frac{5}{2}\), −\(\frac{8}{3}\)

Points

1. Now that you have studied absolute values, you can think that negative numbers become smaller as their absolute values increase.
2. An absolute value can also be regarded as the result of removing a sign from a positive or negative number.
3. The absolute value of 0 is 0.
1. Express the following using a number with a + or − sign.

(1) When "5 points ahead" is expressed as +5 points, how do you say "3 points behind"?

(2) When the point 7 km away to the west is expressed as −7 km, how do you say the point 4 km away to the east?

(3) When "6 pieces left over" is expressed as +6, how do you say "8 pieces short"?

2. On the following number line, what number do points A, B, C, D, and E stand for?

![Number line with points A, B, C, D, and E marked]

3. Express the relative size between the numbers below with an inequality sign.

(1) −10, +3

(2) −4.5, +3.6

(3) −2, −2\frac{3}{4}

4. Answer the following questions about the numbers shown below.

+0.07, +2\frac{1}{3}, −0.5, 0, −8, +5.3, −13, −2\frac{2}{5}, 6

(1) Which are natural numbers?

(2) Which is the greatest negative number?

(3) Arrange the numbers in ascending order of size.

(4) Arrange the numbers in ascending order of absolute value.

5. Answer the following questions.

(1) List all the numbers whose absolute values are 5.

(2) How many integers have absolute values smaller than 6?

6. A factory's target of daily output is 1000 pieces a day. The output from Monday to Friday is shown in the table below. Express the differences in amount between the daily output and the target using a number with a + or − sign, which shows that the daily output is more or less than the target respectively.

<table>
<thead>
<tr>
<th>Daily output</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference between the daily output and the target (1000)</td>
<td>990</td>
<td>1012</td>
<td>1026</td>
<td>1004</td>
<td>987</td>
</tr>
<tr>
<td>Difference</td>
<td></td>
<td>+12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Unit: piece)
Let's learn the basics

1. Addition (1)

The result of addition is the sum.

(1) How to calculate \((+5) + (+2)\)

Go 5 from 0 in the positive direction on the number line, and find the point 2 away from there in the positive direction.

\((+5) + (+2) = +7\)

(2) How to calculate \((+4) + (-6)\)

Go 4 from 0 in the positive direction on the number line, and find the point 6 away from there in the negative direction.

\((+4) + (-6) = -2\)

(3) How to calculate \((-7) + (+3)\)

Go 7 from 0 in the negative direction on the number line, and find the point 3 away from there in the positive direction.

\((-7) + (+3) = -4\)

(4) How to calculate \((-2) + (-4)\)

Go 2 from 0 in the negative direction on the number line, and find the point 4 away from there in the negative direction.

\((-2) + (-4) = -6\)

1. Calculate using the number line.

- (1) \((+3) + (+6)\)
- (2) \((+1) + (+5)\)
- (3) \((+4) + (+4)\)
- (4) \((+8) + (-5)\)
- (5) \((+2) + (-6)\)
- (6) \((+3) + (-10)\)

2. Calculate using the number line.

- (1) \((-1) + (+5)\)
- (2) \((-5) + (+3)\)
- (3) \((-4) + (+4)\)
- (4) \((-2) + (-3)\)
- (5) \((-4) + (-5)\)
- (6) \(0 + (-8)\)
## Let's learn the basics

2 Addition (2)

1. The sum of two numbers with the same sign is the sum of their absolute values with the common sign.
2. The sum of two numbers with different signs is the difference of their absolute values with the sign of the number whose absolute value is greater than the other.
3. When adding 0 to a certain number or add it to 0, both results are the original number. \( a+0=a \), \( 0+a=a \)
4. The sum of two numbers with the same absolute value and different signs is 0.

\[ \begin{align*}
\text{(1)} \quad (-7) + (-9) &= -(7 + 9) = -16 \\
\text{(2)} \quad (+8) + (-13) &= -(13 - 8) = -5 \\
\text{(3)} \quad (-0.9) + (-1.5) &= -(0.9 + 1.5) = -2.4 \\
\text{(4)} \quad (-\frac{2}{7}) + \left( \frac{6}{7} \right) &= \frac{4}{7}
\end{align*} \]

### 3 Calculate.

\[ \begin{align*}
\text{(1)} \quad (+3) + (+5) & \quad \text{(2)} \quad (-3) + (-2) & \quad \text{(3)} \quad (-4) + (-6) \\
\text{(4)} \quad 0 + (+7) & \quad \text{(5)} \quad (+5) + (+9) & \quad \text{(6)} \quad (-8) + (-8) \\
\text{(7)} \quad (+8) + (+13) & \quad \text{(8)} \quad (-24) + (-19) & \quad \text{(9)} \quad (-15) + 0
\end{align*} \]

### 4 Calculate.

\[ \begin{align*}
\text{(1)} \quad (+7) + (-4) & \quad \text{(2)} \quad (-4) + (+9) & \quad \text{(3)} \quad (+3) + (-10) \\
\text{(4)} \quad (+15) + (-9) & \quad \text{(5)} \quad (-13) + (+7) & \quad \text{(6)} \quad 0 + (-12) \\
\text{(7)} \quad (+13) + (-18) & \quad \text{(8)} \quad (-23) + (+42) & \quad \text{(9)} \quad (-26) + (+26)
\end{align*} \]

### 5 Calculate.

\[ \begin{align*}
\text{(1)} \quad (+0.3) + (+5) & \quad \text{(2)} \quad (-3.5) + (-4.3) \\
\text{(3)} \quad (-2.7) + (+1.9) & \quad \text{(4)} \quad \left( -\frac{1}{3} \right) + \left( -\frac{2}{3} \right) \\
\text{(5)} \quad \left( +\frac{3}{5} \right) + \left( -\frac{1}{3} \right) & \quad \text{(6)} \quad \left( -\frac{5}{6} \right) + \left( +\frac{2}{3} \right)
\end{align*} \]
Let's learn the basics  3  Laws of addition

1. **The commutative law of addition** When adding two numbers, you can get the same result by changing their order.
   \[ a + b = b + a \]

2. **The associative law of addition** When adding numbers, you can get the same result whichever pair you start to add.
   \[ (a + b) + c = a + (b + c) \]

3. By using the laws of addition, you can first add only positive numbers, then add only negative numbers, and finally add both.

<table>
<thead>
<tr>
<th>Example</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ((-3) + (+8) + (-1))</td>
<td>[\begin{align*} = (+8) + (-3) + (-1) &amp; \quad \text{commutative law} \ = (+8) + (-4) &amp; \quad \text{associative law} \ = +4 &amp; \end{align*}]</td>
<td>+4</td>
</tr>
<tr>
<td>(2) ((+6) + (-19) + (+12) + (-5))</td>
<td>[\begin{align*} = (+6) + (+12) + (-19) + (-5) &amp; \quad \text{commutative law} \ = (+6) + (+12) + (-19) + (-5) &amp; \quad \text{associative law} \ = (+18) + (-24) &amp; \end{align*}]</td>
<td>-6</td>
</tr>
</tbody>
</table>

6  Calculate.

| (1) \((+5) + (-3) + (-6)\) | (2) \((-4) + (+9) + (-2)\) |
| (3) \((-13) + (-7) + (+8)\) | (4) \((-11) + (+15) + (-6)\) |
| (5) \((-4) + (+5) + 0 + (+12)\) | (6) \((-12) + (+21) + (-15) + (+34)\) |
| (7) \((+19) + (-24) + (-51) + (+37)\) | (8) \((+69) + (-17) + (-26) + (+18)\) |
| (9) \((-9) + (+33) + (+26) + (-18)\) | (10) \((-152) + (+201) + (-103) + (+273)\) |

7  Calculate.

| (1) \((-0.6) + (+0.3) + (-0.7)\) | (2) \((+5.7) + (-4.8) + (+8.2)\) |
| (3) \((+0.75) + (-0.28) + (-0.63)\) | (4) \((+0.9) + (-2.5) + (+1.3) + (-0.8)\) |

8  Calculate.

| (1) \(\left(\frac{1}{7}\right) + \left(-\frac{4}{7}\right) + \left(-\frac{2}{7}\right)\) | (2) \(\left(-\frac{2}{3}\right) + \left(+\frac{3}{5}\right) + \left(+\frac{4}{15}\right)\) |
| (3) \(\left(-\frac{3}{8}\right) + \left(+\frac{1}{4}\right) + \left(+\frac{1}{2}\right) + \left(-\frac{3}{4}\right)\) | (4) \(\left(-\frac{3}{4}\right) + \left(+\frac{1}{3}\right) + \left(+\frac{1}{2}\right) + \left(-\frac{2}{3}\right)\) |
Chapter 1  Signed numbers

Let's learn the basics

1. The result of subtraction is the difference.
2. Subtracting a positive or negative number is the same as adding the same number with the different sign.
3. Subtracting a number from 0 results in the same number with the different sign. Subtracting 0 from a number results in the original number. \(0-a=-a, a-0=a\)

\[(1)\ (-5) - (+8)\] Change the sign of 8 and use addition.
\[= (-5) + (-8)\]
\[= -13\]

\[(2)\ (-3) - (-9)\] Change the sign of -9 and use addition.
\[= (-3) + (+9)\]
\[= +6\]

\[(3)\ (+2.7) - (+3.1)\]
\[= (+2.7) + (-3.1)\]
\[= -0.4\]

\[(4)\ \left(-\frac{4}{5}\right) - \left(-\frac{1}{5}\right)\]
\[\frac{-4}{5} + \frac{1}{5} \]
\[= -\frac{3}{5}\]

9 Calculate.

\[\square (1)\ (+5) - (+2)\]
\[\square (2)\ (-4) - (+7)\]
\[\square (3)\ (-9) - (-4)\]

\[\square (4)\ (+6) - (-8)\]
\[\square (5)\ (-10) - (-3)\]
\[\square (6)\ (+7) - (+16)\]

\[\square (7)\ (-14) - (-18)\]
\[\square (8)\ (+15) - (+29)\]
\[\square (9)\ 0 - (-21)\]

\[\square (10)\ (-16) - (+21)\]
\[\square (11)\ (+33) - (-41)\]
\[\square (12)\ (-48) - 0\]

10 Calculate.

\[\square (1)\ (+0.8) - (-0.3)\]
\[\square (2)\ (-1.7) - (+0.9)\]
\[\square (3)\ (-3.5) - (-1.9)\]

\[\square (4)\ (-3.6) - (+0.8)\]
\[\square (5)\ (+6.3) - (-2.1)\]
\[\square (6)\ (+4.8) - (+6.2)\]

11 Calculate.

\[\square (1)\ \left(+\frac{4}{9}\right) - \left(-\frac{5}{9}\right)\]
\[\square (2)\ \left(+\frac{3}{4}\right) - \left(+\frac{2}{3}\right)\]
\[\square (3)\ \left(-\frac{5}{6}\right) - \left(+\frac{1}{2}\right)\]

\[\square (4)\ \left(-\frac{1}{6}\right) - \left(+\frac{3}{8}\right)\]
\[\square (5)\ \left(+\frac{7}{9}\right) - \left(-\frac{1}{3}\right)\]
\[\square (6)\ \left(-\frac{7}{12}\right) - \left(-\frac{5}{8}\right)\]
### Calculations with both addition and subtraction (1)

1. When using both addition and subtraction, first change the expression to addition-only.
2. When the calculation result is a positive number, the + sign can be removed.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Change to addition-only.</th>
<th>Find the sums of the positive numbers and negative numbers separately.</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+8) - (+4) + (-7) - (-9)</td>
<td>= (+8) + (-4) + (-7) + (+9)</td>
<td>= (+17) + (-11)</td>
<td>= 6</td>
</tr>
</tbody>
</table>

Remove the + sign when the calculation result is a positive number.

12 Calculate.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+2) - (-8) + (-5)</td>
<td>(-6) + (-3) - (-17)</td>
</tr>
<tr>
<td>(-9) - (-7) + (+13)</td>
<td>(-20) - (-4) + (-8)</td>
</tr>
<tr>
<td>(-4) + (-12) - (+16)</td>
<td>(+19) - (+25) + (+11)</td>
</tr>
<tr>
<td>(-9) + (-4) - (+15) + (+23)</td>
<td>(-8) + (+4) - (-10) + (+7)</td>
</tr>
<tr>
<td>(-7) - (+12) - (-28) + (-5)</td>
<td>(+13) - (+6) + (-4) - (+9)</td>
</tr>
<tr>
<td>(+12) - (+13) - (-6) + (-21)</td>
<td>(-26) - (-35) + (-18) + (+23)</td>
</tr>
</tbody>
</table>

13 Calculate.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.8) - (+0.5) + (-0.3)</td>
<td>(+1.9) - (+1.2) + (-2.6)</td>
</tr>
<tr>
<td>(-0.35) + (-0.17) - (-0.51)</td>
<td>(+0.9) - (+0.3) - (-1.8) + (-0.6)</td>
</tr>
<tr>
<td>(+2.2) - (-6) + (-9.8) - (+0.5)</td>
<td>(-2.36) - (-3.9) - (-2.81) + (+0.3)</td>
</tr>
</tbody>
</table>

14 Calculate.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+2/5) + (-5/9) - (-1/9)</td>
<td>(-7/15) - (-11/15) + (-13/15)</td>
</tr>
<tr>
<td>(+2/3) + (-1/2) - (-5/6)</td>
<td>(-1/8) - (-7/8) + (-3/8) + (5/8)</td>
</tr>
<tr>
<td>(-3/5) + (-11/15) - (-4/3) + (1/3)</td>
<td>(-5/9) - (-2/3) + (-1/6) - (1/2)</td>
</tr>
</tbody>
</table>
## Let's Learn the Basics

### 6. Calculations with both addition and subtraction (2)

When using both addition and subtraction, you can remove the parentheses by changing the expression to addition-only and then remove the + signs of addition.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Calculation Steps</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ((-9) - (-6) + (-8))</td>
<td>Change to addition-only. Remove the + signs and parentheses. Change the order and find the sum of the positive terms and negative terms separately.</td>
<td>(-11)</td>
</tr>
<tr>
<td>(2) (-16 - (-23) + 4 + (-19))</td>
<td></td>
<td>(-8)</td>
</tr>
</tbody>
</table>

(Note) In an expression like \(6-9+5\), 6, -9, and 5 are called the terms of this expression. 6 and 5 are positive terms, and -9 is a negative term.

### 15 Remove the parentheses and calculate.

<table>
<thead>
<tr>
<th>No.</th>
<th>Expression</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>((+4) + (-3) - (-12))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>((-5) + (-6) - (-4))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>((+7) + (-6) + (-2) - (-3))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>((-19) + (-15) - (-13) + (+8))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>((+15) - (-4) + (-27) - (-12))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td>((-18) - (-36) + (+7) + (-11))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 16 Calculate.

<table>
<thead>
<tr>
<th>No.</th>
<th>Expression</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(9 - 5 - 6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td></td>
<td></td>
<td>(-2)</td>
</tr>
<tr>
<td>(3)</td>
<td>(-30 - 9 - (-19) + 25)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>(48 + (-29) - 58 - (-38))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>(-17 - (-25) + 16 - 19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td>(32 + (-47) - 16 + 22)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7)</td>
<td>(24 - (+49) - (-14) + 15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(8)</td>
<td>(-29 + 35 - (-17) - 42)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 17 Calculate.

<table>
<thead>
<tr>
<th>No.</th>
<th>Expression</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(0.9 - 1 - 0.5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>(1.5 - 2.3 - 0.8 + 0.7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>(3.8 - 1.5 - (-4.6) + (-2.7))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>(-2.15 + 3.24 + (-1.08) - (-3.91))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 18 Calculate.

<table>
<thead>
<tr>
<th>No.</th>
<th>Expression</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(\frac{2}{3} \cdot \frac{1}{4} \div \frac{5}{6})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td>(\frac{3}{10} + \frac{1}{15} - \frac{4}{5})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)</td>
<td>(\frac{1}{2} + \left(\frac{2}{7} \div \frac{4}{7}\right) - \frac{3}{14})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4)</td>
<td>(-\frac{3}{4} - \left(-\frac{1}{3}\right) + \frac{7}{12} + \left(-\frac{1}{2}\right))</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

20
<table>
<thead>
<tr>
<th>Exercises</th>
<th>2 Addition and subtraction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong> Calculate.</td>
<td></td>
</tr>
<tr>
<td>□(1) ((-3) + (-12))</td>
<td>□(2) ((-10) + (+6))</td>
</tr>
<tr>
<td>□(4) ((-24) + (+15))</td>
<td>□(5) ((+13) + (-32))</td>
</tr>
<tr>
<td>□(7) ((+5.6) + (-3.9))</td>
<td>□(8) ((-\frac{5}{7}) + (-\frac{1}{7}))</td>
</tr>
<tr>
<td><strong>2</strong> Calculate.</td>
<td></td>
</tr>
<tr>
<td>□(1) ((+9) + (-14) + (+2))</td>
<td>□(2) ((-15) + (+6) + (-8) + (+9))</td>
</tr>
<tr>
<td>□(3) ((-0.8) + (+1.2) + (-2.6))</td>
<td>□(4) ((+2) + (+3.8) + (-1.6) + (-1))</td>
</tr>
<tr>
<td>□(5) ((-\frac{3}{8}) + (+\frac{5}{8}) + (-\frac{7}{8}))</td>
<td>□(6) ((-\frac{1}{2}) + (+\frac{1}{3}) + (-\frac{1}{4}) + (+\frac{1}{6}))</td>
</tr>
<tr>
<td><strong>3</strong> Calculate.</td>
<td></td>
</tr>
<tr>
<td>□(1) ((+7) - (-13))</td>
<td>□(2) ((-21) - (-13))</td>
</tr>
<tr>
<td>□(4) (17 - (+25))</td>
<td>□(5) (-39 - (-54))</td>
</tr>
<tr>
<td>□(7) (-0.55 - 0.19)</td>
<td>□(8) ((-\frac{3}{8}) - \frac{7}{8})</td>
</tr>
<tr>
<td><strong>4</strong> Calculate.</td>
<td></td>
</tr>
<tr>
<td>□(1) ((-10) + (+5) - (-9) - (+8))</td>
<td>□(2) ((-26) - (+3) - (-18))</td>
</tr>
<tr>
<td>□(3) (-11 - 0 - (-19) - 16)</td>
<td>□(4) (-15 - 9 + 14 + 12)</td>
</tr>
<tr>
<td>□(5) (-0.46 + 1.6 - (+0.78))</td>
<td>□(6) ((-2.3) + 5.7 + (-1.1) + 3.8)</td>
</tr>
<tr>
<td>□(7) (\frac{1}{2} - (-\frac{1}{4}) + (-\frac{1}{3}) + \frac{1}{6})</td>
<td>□(8) (-\frac{1}{3} + (-\frac{7}{4}) - (-\frac{1}{2}) + \frac{4}{5})</td>
</tr>
</tbody>
</table>
Let's learn the basics.  

1. Multiplication

The result of multiplication is the **product**.

2. The product of two numbers with the same sign

- Put a positive sign on the product of their absolute values: 
  
  $$(+) \times (+) \rightarrow (+), \quad (-) \times (-) \rightarrow (+)$$

3. The product of two numbers with different signs

- Put a negative sign on the product of their absolute values: 
  
  $$(+) \times (-) \rightarrow (-), \quad (-) \times (+) \rightarrow (-)$$

4. $a \times (-1) = -a$, $(-1) \times a = -a$

5. $a \times 0 = 0, \quad 0 \times a = 0$

### Examples

1. $(-3) \times (-6)$ — The product of two numbers with the same sign is the product of their absolute values with a positive sign.

   - $+ (3 \times 6)$
   - $= +18$
   - $= -18$

2. $(-7) \times (+4)$ — The product of two numbers with different signs is the product of their absolute values with a negative sign.

   - $- (7 \times 4)$
   - $= -28$

### Practice Problems

1. Calculate.

   - $\square (1) \quad (+3) \times (+7)$
   - $\square (2) \quad (-4) \times (-9)$
   - $\square (3) \quad (-9) \times 0$
   - $\square (4) \quad (-23) \times (-1)$
   - $\square (5) \quad (-30) \times (-12)$
   - $\square (6) \quad (+12) \times (+20)$

2. Calculate.

   - $\square (1) \quad (+4) \times (-8)$
   - $\square (2) \quad (-6) \times (+9)$
   - $\square (3) \quad (-7) \times (+5)$
   - $\square (4) \quad 0 \times (-6)$
   - $\square (5) \quad (+12) \times (-1)$
   - $\square (6) \quad (+15) \times (-8)$
   - $\square (7) \quad (-5) \times (+16)$
   - $\square (8) \quad (+25) \times (-14)$
   - $\square (9) \quad (-32) \times (+18)$

3. Calculate.

   - $\square (1) \quad (-0.4) \times (-7)$
   - $\square (2) \quad (+0.8) \times (-0.3)$
   - $\square (3) \quad (-5.2) \times (-0.4)$
   - $\square (4) \quad (-\frac{1}{4}) \times (+\frac{2}{3})$
   - $\square (5) \quad (-\frac{5}{6}) \times (+\frac{2}{15})$
   - $\square (6) \quad (-\frac{7}{24}) \times (-\frac{4}{21})$
   - $\square (7) \quad (-\frac{3}{8}) \times (+\frac{8}{5})$
   - $\square (8) \quad (-\frac{7}{6}) \times (-\frac{15}{14})$
   - $\square (9) \quad (+\frac{21}{8}) \times (-\frac{32}{9})$
Let's learn the basics  

1. The sign of the product of some numbers is determined by the number of negative terms in the expression: Odd number → Negative sign (−), Even number → Positive sign (+)

2. The commutative law of multiplication  
\[ a \times b = b \times a \]

   The associative law of multiplication  
\[ (a \times b) \times c = a \times (b \times c) \]

   
   \[ (1) \quad (-3) \times (-5) \times (-4) \]
   
   Since there are an odd number of negative terms in the expression, put a negative sign on the product of their absolute values.
   
   \[ = - (3 \times 5 \times 4) \]
   
   \[ = -60 \]

   
   \[ (2) \quad (-2) \times (+3) \times (-8) \]
   
   Since there are an even number of negative terms in the expression, put a positive sign on the product of their absolute values.
   
   \[ = + (2 \times 3 \times 8) \]
   
   \[ = 48 \]

3. Applying the commutative law and the associative law
\[ (-2.5) \times (+2.3) \times (-2) \]

   The commutative law
   
   \[ = (-2.5) \times (-2) \times (+2.3) \]
   
   The associative law
   
   \[ = 5 \times (+2.3) \]
   
   \[ = 11.5 \]

4. Calculate.

   □(1)  \(-6) \times (+7) \times (-2)\]
   □(2)  \((+9) \times (-3) \times (+7)\]
   □(3)  \((-4) \times (+2) \times (-7) \times (+5)\]
   □(4)  \(2 \times (-8) \times 3 \times (-6)\]
   □(5)  \((-4) \times 6 \times (-5) \times (-3)\]
   □(6)  \((-1) \times 15 \times 3 \times 6\]

5. Calculate efficiently.

   □(1)  \((-0.7) \times (-8) \times (-10)\]
   □(2)  \((+0.2) \times (-3.7) \times (-5)\]
   □(3)  \((-1.2) \times (+7) \times (-5)\]
   □(4)  \((-5) \times (-9.2) \times (-0.8)\]
   □(5)  \((-25) \times (-13) \times (+4)\]
   □(6)  \((-1.25) \times (+39) \times (+8)\]

6. Calculate.

   □(1)  \((-\frac{2}{3}) \times (-\frac{3}{4}) \times (-\frac{2}{5})\]
   □(2)  \((-\frac{1}{2}) \times (+\frac{5}{6}) \times (-8)\]
   □(3)  \((-\frac{2}{3}) \times (-6) \times (-\frac{5}{9})\]
   □(4)  \(1\frac{3}{5} \times (-\frac{3}{4}) \times \frac{5}{12}\]
   □(5)  \((-\frac{3}{4}) \times (-2) \times \frac{2}{15} \times (-\frac{5}{3})\]
   □(6)  \(\frac{7}{12} \times (-1\frac{1}{4}) \times 6 \times (-\frac{3}{8})\]
Chapter 1  Signed numbers

Let's learn the basics  3  Powers and exponents

The product of a number multiplied by itself is called powers, and the small number to the upper right is called an exponent.

1. $5 \times 5$ is written as $5^2$ and read as **5 to the second power** or **5 squared**.
   $5 \times 5 \times 5$ is written as $5^3$ and read as **5 to the third power** or **5 cubed**.

2. Don't forget to put parentheses around the powers of a negative number or a fraction. Also, pay attention to the difference between $(-3)^2$ and $-3^2$.

### (1) How to use exponents of powers

1. Write a small number to the upper right to show how many times the number is multiplied by itself.
   
   $4 \times 4 \times 7 \times 7 = 4^2 \times 7^2$

2. Put parentheses around the fraction.
   
   $\left( \frac{1}{2} \right) \times \left( \frac{1}{2} \right) \times \left( \frac{1}{2} \right) = \left( \frac{1}{2} \right)^3$

### (2) Calculations with powers

1. $(-5)^2$ is $-5$ multiplied twice by itself.
   
   $(-5) \times (-5) = 25$

2. $-5^2$ is $5^2$ with a negative sign.
   
   $-(5 \times 5) = -25$

---

7  Express the following expressions using exponents of powers.

- $5 \times 5 \times 5$
- $2 \times 2 \times 2 \times 3 \times 3$
- $(-3) \times (-3) \times (-3) \times 8$
- $4 \times 4 \times (-5) \times (-5) \times (-5)$
- $\left( \frac{-2}{3} \right) \times \left( \frac{-2}{3} \right) \times \left( \frac{-2}{3} \right)$
- $\frac{3}{4} \times \frac{3}{4} \times \left( \frac{-5}{6} \right) \times \left( \frac{-5}{6} \right)$

8  Calculate.

- $2^4$
- $(-8)^2$
- $-8^2$
- $(-1)^5$
- $(2 \times 5)^4$
- $(-7) \times (-2)^3$
- $(-1)^3 \times 6$
- $2^2 \times (-3)^2$
- $9^2 \times (-2)^3$
- $(-3)^2 \times (-10)^2$
- $\left( \frac{-1}{4} \right)^2$
- $\left( \frac{-3}{5} \right)^3$
Let's learn the basics 4 Division (1)

1. The result of division is the quotient.
2. The quotient of two numbers with the same sign
   → Put a positive sign on the quotient of their absolute values $(+) ÷ (+) → (+), (-) ÷ (-) → (+)$
3. The quotient of two numbers with different signs
   → Put a negative sign on the quotient of their absolute values $(+) ÷ (-) → (-), (-) ÷ (+) → (-)$
4. 0 divided by any number is $0 ÷ a = 0$. No numbers can be divided by 0.

1) $(-24) ÷ (-3)$ The quotient of two numbers with the same sign is the quotient of their absolute values with a positive sign.

\[ \begin{align*}
\text{(1)} & \quad (-24) ÷ (+3) = 8 \\
\text{(2)} & \quad (-28) ÷ (+4) = 7
\end{align*} \]

9 Calculate.

\[ \begin{align*}
\text{□(1)} & \quad (+18) ÷ (+3) \\
\text{□(2)} & \quad (-15) ÷ (-5) \\
\text{□(3)} & \quad (-18) ÷ (+2) \\
\text{□(4)} & \quad (-21) ÷ (-7) \\
\text{□(5)} & \quad (+63) ÷ (-9) \\
\text{□(6)} & \quad 13 ÷ (-1) \\
\text{□(7)} & \quad (-96) ÷ (-8) \\
\text{□(8)} & \quad (-144) ÷ 6 \\
\text{□(9)} & \quad 125 ÷ (-25) \\
\text{□(10)} & \quad 0 ÷ (-8) \\
\text{□(11)} & \quad (-99) ÷ 11 \\
\text{□(12)} & \quad (-135) ÷ (-15) \\
\text{□(13)} & \quad 189 ÷ (-7) \\
\text{□(14)} & \quad (-57) ÷ 19 \\
\text{□(15)} & \quad -345 ÷ (-23)
\end{align*} \]

10 Calculate.

\[ \begin{align*}
\text{□(1)} & \quad (-1.2) ÷ (-6) \\
\text{□(2)} & \quad (+4.2) ÷ (-7) \\
\text{□(3)} & \quad (-5.4) ÷ 9 \\
\text{□(4)} & \quad (-0.36) ÷ 4 \\
\text{□(5)} & \quad (-7.2) ÷ (-0.8) \\
\text{□(6)} & \quad 12.8 ÷ (-0.4)
\end{align*} \]

11 Fill in [ ] with a suitable number.

\[ \begin{align*}
\text{□(1)} & \quad (-4) \times \Box = 56 \\
\text{□(2)} & \quad (-3) \times \Box = -4.8 \\
\text{□(3)} & \quad \Box \times (-5) = 60 \\
\text{□(4)} & \quad (-6) \times \Box \times (-1) = -42 \\
\text{□(5)} & \quad (-8) \times \Box \times 3 = -120 \\
\text{□(6)} & \quad (-0.2) \times \Box \times (-4) = -6.4
\end{align*} \]
Let's learn the basics | Division (2) Using reciprocals

When the product of two numbers is 1, one of them is called the **reciprocal** of the other.

**Examples**

1. The reciprocal of \( \frac{3}{4} \) is \( \frac{4}{3} \).
2. The reciprocal of \( -\frac{1}{5} \) is \( -5 \).
3. 0.6 is equal to \( \frac{3}{5} \), so its reciprocal is \( \frac{5}{3} \).

Dividing by a positive or negative number is the same as multiplying by its reciprocal.

\[
\begin{align*}
(1) \quad (-6) \div (-8) &= (-6) \times \left( -\frac{1}{8} \right) \\
&= \frac{3}{4} \\
&\text{Multiply by the reciprocal of the divisor.}
\end{align*}
\]

12 Calculate using reciprocals.

\[
\begin{align*}
\text{(1)} \quad 3 \div (-6) \\
\text{(2)} \quad (-2) \div 8 \\
\text{(3)} \quad (-5) \div (-15) \\
\text{(4)} \quad (-10) \div (-25) \\
\text{(5)} \quad (-18) \div 16 \\
\text{(6)} \quad (-28) \div (-36)
\end{align*}
\]

13 Calculate.

\[
\begin{align*}
\text{(1)} \quad \left( -\frac{2}{5} \right) \div (-3) \\
\text{(2)} \quad \left( -\frac{4}{7} \right) \div 6 \\
\text{(3)} \quad \frac{3}{8} \div (-9) \\
\text{(4)} \quad -3 \div \left( -\frac{1}{4} \right) \\
\text{(5)} \quad 8 \div \left( -\frac{2}{5} \right) \\
\text{(6)} \quad (-12) \div \left( -\frac{3}{4} \right) \\
\text{(7)} \quad 6 \div \left( -\frac{4}{7} \right) \\
\text{(8)} \quad (-15) \div \left( -\frac{20}{7} \right) \\
\text{(9)} \quad (-28) \div \frac{16}{9}
\end{align*}
\]

14 Calculate.

\[
\begin{align*}
\text{(1)} \quad \left( -\frac{1}{3} \right) \div \frac{1}{2} \\
\text{(2)} \quad \left( -\frac{2}{5} \right) \div \left( -\frac{1}{3} \right) \\
\text{(3)} \quad \frac{5}{6} \div \left( -\frac{1}{4} \right) \\
\text{(4)} \quad \frac{8}{21} \div \left( -\frac{4}{7} \right) \\
\text{(5)} \quad \left( -\frac{8}{15} \right) \div \left( -\frac{12}{25} \right) \\
\text{(6)} \quad \left( -\frac{24}{35} \right) \div \frac{9}{14} \\
\text{(7)} \quad \left( -\frac{2}{3} \right) \div \frac{5}{6} \\
\text{(8)} \quad \left( -\frac{4}{15} \right) \div \left( -2\frac{2}{5} \right) \\
\text{(9)} \quad 3\frac{3}{8} \div \left( -2\frac{1}{4} \right)
\end{align*}
\]
1. Let's learn the basics

When using both multiplication and division, use reciprocals to change the expression to multiplication-only.

2. When there are terms using powers, calculate them first.

\[
\begin{align*}
(1) & \quad 3 \div (-6) \times (-24) \div (-2) \\
& = 3 \times \left(\frac{-1}{6}\right) \times (-24) \times \left(\frac{-1}{2}\right) \\
& = \left(3 \times \frac{-1}{6} \times 24 \times \frac{-1}{2}\right) \\
& = -6
\end{align*}
\]

\[
\begin{align*}
(2) & \quad (-2)^3 \div (-6)^3 \\
& = -8 \div (-36) \\
& = -8 \times \left(\frac{1}{36}\right) \\
& = -\left(8 \times \frac{1}{36}\right) \\
& = \frac{-2}{9}
\end{align*}
\]

15 Calculate.

\[
\begin{align*}
(1) & \quad (-24) \times 3 \div (-6) & (2) & \quad 4 \div (-7) \times 28 \\
(3) & \quad (-8) \div (-12) \times (-15) & (4) & \quad (-30) \times 2 \div (-5) \div (-3) \\
(5) & \quad 72 \div (-18) \times (-3) \div 4 & (6) & \quad (-108) \div (-9) \times 5 \div (-6)
\end{align*}
\]

16 Calculate.

\[
\begin{align*}
(1) & \quad (-4)^3 \div (-2) & (2) & \quad (-3)^3 \div (-2)^3 \\
(3) & \quad 6 \times (-3)^2 \div 27 & (4) & \quad 3^2 \times (-5) \div (-6)^2 \\
(5) & \quad (-2)^3 \times 12 \div (-4)^3 \div 3 & (6) & \quad (-2)^3 \div (-6) \div 18 \times (-3)^2
\end{align*}
\]

17 Calculate.

\[
\begin{align*}
(1) & \quad \left(-\frac{2}{3}\right) \div \left(-\frac{1}{3}\right) \times \frac{1}{6} & (2) & \quad \frac{5}{6} \times \left(-\frac{4}{5}\right) \div \frac{2}{3} \\
(3) & \quad \left(\frac{1}{3}\right)^3 \times \left(-\frac{3}{4}\right) \div \frac{1}{24} & (4) & \quad -\frac{1}{8} \div (-0.5)^3 \times (-4)^2 \\
(5) & \quad 1 \frac{2}{5} \div \left(-2 \frac{4}{7}\right) \times \frac{3}{5} \times \left(-1 \frac{2}{7}\right) & (6) & \quad -\frac{5}{8} \div 0.25 \times (-0.2)^3 \div \left(-\frac{2}{5}\right)^3
\end{align*}
\]

Point

1. When there are decimals in the expression, change them to fractions first.
Let's learn the basics

Calculations with the four arithmetic calculations

1. Addition, subtraction, multiplication, and division are called the **four arithmetic calculations**.
2. The order of calculations with the four arithmetic calculations is as follows:
   - Powers → Inside the parentheses → Multiplication and division → Addition and subtraction

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Steps</th>
</tr>
</thead>
</table>
| (1) \(-5\) + 3 \times (-4) | \(-5\) + (-12) \[ Multiplication first \] 
| = \(-17\) | \(-17\) \[ Addition second \] |
| | |
| (2) \(10 - (5 - 3^2) \div 2\) | \(10 - (5 - 9) \div 2\) \[ Calculation with powers \] 
| = 10 - (-9) \div 2 | = 10 - (-4) \div 2 \[ Calculation inside the parentheses \] 
| = 10 - (-2) | = 10 - (-2) \[ Division \] 
| = 12 | = 12 \[ Subtraction \] |

18 Calculate.

1. \(7 \times (-8) + 15\)
2. \(21 - 12 \div (-3)\)
3. \(15 - (-4) \times (-6)\)
4. \((-3) \times 6 - (-8) \div 4\)
5. \((-36 \div (-9) + 7 \times (-6)\)
6. \(8 + 2 \times (2 - 5)\)
7. \(56 \div (3 - 7) - 5\)
8. \((-7) \times (4 - 9) - (-7)\)
9. \((-3) \times (-5 - (-9)) - 19\)
10. \(13 - \{9 \div (2 - 5)\} \times (-5)\)

19 Calculate.

1. \(2 \times (-3)^2 + 6 \times (-4)\)
2. \((-2)^3 \times 3 - 4^2 \div 8\)
3. \((-2)^2 \div 4 - (-3)^2 \times 5\)
4. \((-4)^3 \div (-2) - 7 \div (-1)^2\)
5. \(3 + (-5)^2 \div (-4) - (-3)^2\)
6. \((-12) \times (40 - 6^2) - (-1)^5 \times 3\)

20 Calculate.

1. \(-5.2 + 3.6 \div (-0.4)\)
2. \((0.25 - (-0.5)) \times (-0.6)\)
3. \(\frac{3}{5} \times \left(\frac{-4}{5}\right)\)
4. \((-\frac{3}{8}) \div \left(\frac{-3}{2}\right) - \frac{3}{4}\)
5. \(\frac{1}{12} \times (-3) - 6 \div \left(\frac{-2}{3}\right)\)
6. \((-\frac{3}{4}) \div (-1.5) + \left(\frac{-1}{2}\right) \times 2.25\)
Let's learn the basics | Making calculations easy — Using the distributive law

The following is true of signed numbers. It is called the **distributive law**.

\[ a \times (b+c) = a \times b + a \times c, \quad (a+b) \times c = a \times c + b \times c \]

As shown below, you can sometimes calculate easily using the distributive law.

### 21 Calculate using the distributive law.

1. \( 12 \times \left( \frac{2}{3} - \frac{3}{4} \right) \)
   
   \[ = 12 \times \frac{2}{3} - 12 \times \frac{3}{4} \]
   
   \[ = 8 - 9 \]
   
   \[ = -1 \]

2. \( (\frac{1}{6} + \frac{2}{3}) \times \frac{1}{18} \)
   
   \[ = \left( \frac{1}{6} + \frac{2}{3} \right) \times 18 \]
   
   \[ = \frac{1}{6} \times 18 + \frac{2}{3} \times 18 \]
   
   \[ = 3 + 12 \]
   
   \[ = 15 \]

3. \( (-7) \times 82 + (-7) \times 18 \)
   
   \[ = (-7) \times (82 + 18) \]
   
   \[ = (-7) \times 100 \]
   
   \[ = -700 \]

4. \( 99 \times (-17) \)
   
   \[ = (100 - 1) \times (-17) \]
   
   \[ = 100 \times (-17) - 1 \times (-17) \]
   
   \[ = -1700 + 17 \]
   
   \[ = -1683 \]

### 22 Calculate using the distributive law.

1. \( -60 \times \left( \frac{1}{5} + \frac{7}{12} \right) \)
   
   \( \square \)

2. \( 18 \times \left( \frac{5}{9} - \frac{1}{6} \right) \)
   
   \( \square \)

3. \( \left( \frac{5}{8} - \frac{7}{12} \right) \times 24 \)
   
   \( \square \)

4. \( \left( \frac{3}{4} + \frac{1}{6} \right) \times \frac{1}{12} \)
   
   \( \square \)

5. \( \left( \frac{5}{6} - \frac{7}{8} \right) \times \left( \frac{1}{24} \right) \)
   
   \( \square \)

6. \( \left( \frac{5}{6} - \frac{8}{9} + \frac{7}{12} \right) \times \left( \frac{1}{36} \right) \)
   
   \( \square \)

### 23 Calculate using the distributive law.

1. \( 13 \times (-14) + 13 \times (-36) \)
   
   \( \square \)

2. \( 59 \times 21 + 41 \times 21 \)
   
   \( \square \)

3. \( 3.14 \times 276 - 3.14 \times 76 \)
   
   \( \square \)

4. \( 12.3 \times 0.45 - 2.3 \times 0.45 \)
   
   \( \square \)

5. \( 98 \times 23 \)
   
   \( \square \)

6. \( (-103) \times 78 \)
   
   \( \square \)

7. \( (-45) \times 101 \)
   
   \( \square \)

8. \( 65 \times (-998) \)
   
   \( \square \)
Chapter 1  Signed numbers

Let's learn the basics 1.9  Judging the sign

| Question | If $a \times b < 0$, $a \times c > 0$, and $c > 0$, is $b$ a positive number or a negative number? |
| Solution | Since $a \times c$ is a positive number, $a$ and $c$ have the same sign. $c$ is a positive number, so $a$ is also a positive number. Since $a \times b$ is a negative number, $a$ and $b$ have different signs. $a$ is a positive number, so $b$ is a negative number. |
| Answer | Negative number |

24 Answer the following questions.

☐ (1) If $a \cdot b < 0$, $b \times c > 0$, and $a > 0$, is $c$ a positive number or a negative number?

☐ (2) If $a \times c > 0$, $a + b > 0$, and $c < 0$, is $b$ a positive number or a negative number?

☐ (3) If $a < 0$, $b > 0$, and $c < 0$, is $a \div c + b \times c$ a positive number or a negative number?

25 Answer the following questions.

☐ (1) If $b < a < 0$, which is the smallest number, $a + b$, $a - b$, $a \times b$, or $a \div b$?

☐ (2) If $a > 0$, $b < 0$, and $a + b > 0$, arrange $a$, $b$, $-a$, and $-b$ in ascending order of size.

Let's learn the basics 10  Sets of numbers and the four arithmetic calculations

Sets of numbers we have studied can be shown in the figure on the right.

Let us think whether the four arithmetic calculations ($a + b$, $a - b$, $a \times b$, $a \div b$) can always be possible in a set of numbers. For example, in the set of natural numbers, addition is always possible, while subtraction is not, like $3 - 5 = 2$.

26 In the table below, mark a circle (○) if the calculation is always possible in the set, and mark a cross (×) if it isn’t. Exclude dividing by 0.

<table>
<thead>
<tr>
<th>Set of natural numbers</th>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set of integers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Set of all numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3 Multiplication and division

Exercises

1. Calculate.
   - (1) \((-4) \times 4\)
   - (2) \((-7) \times (-9)\)
   - (3) \(0.6 \times (-12)\)

   - (4) \((-\frac{7}{18}) \times (-\frac{9}{28})\)
   - (5) \((-5)^2 \times (-2)^2\)
   - (6) \((-\frac{2}{3})^3 \times \left(\frac{1}{2}\right)^2\)

   - (7) \((+6) \times (-4) \times (+5)\)
   - (8) \((-1)^3 \times (-2) \times 3^5\)

2. Calculate.
   - (1) \((-42) \div 7\)
   - (2) \((-16) \div (-20)\)
   - (3) \(9 \div \left(-\frac{3}{5}\right)\)

   - (4) \((-\frac{5}{6}) \div \frac{2}{3}\)
   - (5) \(2 \frac{4}{7} \div \left(-1 \frac{2}{7}\right)\)
   - (6) \((-1 \frac{7}{8}) \div (-0.75)\)

3. Calculate.
   - (1) \((-42) \times 2 \div (-7)\)
   - (2) \((-16) \div (-12) \times (-9)\)

   - (3) \(6^2 \div (-2^2) \times (-5)\)
   - (4) \((-\frac{2}{3})^2 \times \left(-\frac{1}{2}\right)^2 \div \frac{1}{16}\)

   - (5) \(-7 + 3 \times (-2)\)
   - (6) \((-54) \div 9 + (-12) \times (-7)\)

   - (7) \(6 + (17 - 4 \times 5) \div 3\)
   - (8) \(\frac{1}{4} + \frac{1}{2} \times \left(-\frac{2}{3}\right)\)

   - (9) \((-\frac{1}{2}) \times 1.2 - 0.25 \div \frac{3}{2}\)
   - (10) \((-\frac{1}{3})^2 \div \left(\frac{1}{2} - 2 \div \frac{2}{3}\right)\)

4. Calculate using the distributive law.
   - (1) \((-72) \times \left(\frac{13}{18} - \frac{5}{12}\right)\)
   - (2) \(48 \times (-73) + 48 \times (-27)\)

5. If \(a > 0,\) \(b < 0,\) and \(c < 0,\) is \(a + b^2 - c^4\) a positive number or a negative number?
Let's learn the basics I Using signed numbers

**Question**

Three people A, B, and C played a game. The sum of their scores was 0. When A got -6 points and B got 14 points, how many points did C get?

**Solution**

Subtract A's score and B's score from the sum. Thus, \(0 - (-6) - 14 = 0 + 6 - 14 = -8\) points.

**Answer**

-8 points

1. Three people A, B, and C played a game. The sum of their scores was 5. Answer the following questions.

   1. When A got 15 points and B got -20 points, how many points did C get?
   
   - Answer: \(-8\) points
   
   2. The average score of B and C was -7 points. How many points did A get?

2. You toss a coin and get 3 points for heads and 2 points for tails. For a game in which you try six times, answer the following questions.

   1. The results were heads, tails, tails, heads, tails, and tails. What is the total score?
   
   - Answer: \(5\) points
   
   2. The total score is 8 points. How many times were heads and tails each?

3. Point P is the origin of the number line on the right.

   It moves according to the number on the face of a die when you cast it: if it is an even number, Point P moves by that number in the positive direction, and if it is an odd number, Point P moves by that number in the negative direction.

   The table on the right shows how many times you got each number on the face of the die after casting it a certain number of times. When Point P is finally at -4, find the number to put in the blank of the table.

   ![Number on the face of a die and Times the die is cast table]

   Number on the face of a die: 1, 2, 3, 4, 5, 6

   Times the die is cast: 2, 2, 1, 2, 1

4. Fill in the blanks of the tables below so that the sum becomes equal when adding any line of three numbers horizontally, vertically, or diagonally.

   **(1)**
   
<table>
<thead>
<tr>
<th>-4</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>

   **(2)**
   
<table>
<thead>
<tr>
<th>7</th>
<th>-4</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>-8</td>
<td>3</td>
<td>-1</td>
</tr>
</tbody>
</table>
Let's learn the basics  2  Signed numbers and tables (1) – Comparison to the standard

<table>
<thead>
<tr>
<th>Question</th>
<th>The table on the right shows the difference between D's height and each height of A to E. Answer the following questions.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>+3</td>
</tr>
<tr>
<td>D's height and each height of A to E. Answer the following questions.</td>
<td>(Unit: cm)</td>
</tr>
<tr>
<td></td>
<td>(1) How many centimeters taller is A than B?</td>
</tr>
<tr>
<td></td>
<td>(2) D is 157 cm tall. Find these five people's average height.</td>
</tr>
<tr>
<td></td>
<td>(Solution) (1) The difference between A's height and B's height is, ((+3) - (-7) = 10) (cm)</td>
</tr>
<tr>
<td></td>
<td>(2) The sum of the differences between D's height and the five people's (including D) is, ((+3) + (-7) + (+1) + 0 + (-2) = -5) (cm)</td>
</tr>
<tr>
<td></td>
<td>Therefore, the five people's average height is, (157 + (-5) ÷ 5 = 156) (cm)</td>
</tr>
<tr>
<td></td>
<td>Answer (1) 10 cm taller (2) 156 cm</td>
</tr>
</tbody>
</table>

5  The table on the right shows the differences between F's score and each score of A to F on a math test. Answer the following questions.

- (1) How many points higher is the highest score than the lowest score?
- (2) F's score is 76 points. Find these six students' average score.

6  The table on the right shows the results of subtracting D's height from each height of A to F using cm as a unit. If their average height is 162 cm, what is F's height?

7  The table below shows A's to F's scores on a Japanese test, using positive or negative numbers to represent higher or lower scores than the standard point respectively. Answer the following questions.

<table>
<thead>
<tr>
<th>Student</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Score) - (Standard point)</td>
<td>+1</td>
<td>-8</td>
<td>+3</td>
<td>+4</td>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>(Unit: points)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- (1) How many points higher is A's score than F's score?
- (2) When the standard point is 84, answer the following questions.
  1. Find the six students' average score.
  2. When the scores of the three students G, H, and I are added to those of the six students A to F, the nine students' average score is 79. Find the average score of G, H, and I.
Chapter 1  Signed numbers

Let's learn the basics  3  Signed numbers and tables (2) – Comparison to the previous time

Question

The table on the right shows A's scores on 5 math tests, using a positive or negative number to represent a higher or lower score than the previous time respectively. When the score of the third test is 75 points, answer the following questions.

(1) How many points is the score of the first test?
(2) Find the average score of the five tests.

Solution

(1) The score of the second test is, 75 - (-6) = 81 (points).
    So, the score of the first test is, 81 - (+8) = 73 (points).

(2) The score of the fourth test is, 75 + (+12) = 87 (points),
    and that of the fifth test is, 87 + (-8) = 79 (points).
    So, the scores of all the tests are those shown on the right.
    The sum of the scores of the five tests is, 73 + 81 + 75 + 87 + 79 = 395 (points).
    Therefore, the average score of the five tests is, 395 ÷ 5 = 79 (points).

Answer

(1) 73 points  (2) 79 points

8  The table below shows the number of computation practice problems someone solved in a week, using a positive or negative number to represent more or fewer problems than the previous day respectively. When this person solved 23 problems on Wednesday, answer the following questions.

<table>
<thead>
<tr>
<th>Day</th>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference in the number of problems compared to the previous day</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

□1) Based on the table above, you can make the following table to show the number of problems this person solved each day. Fill in the blanks.

<table>
<thead>
<tr>
<th>The numbers Of problems</th>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>21</td>
<td>23</td>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

□2) How many problems a day did this person solve on average in the week?

9  The table below shows the temperatures at 10 a.m. in a week, using a positive or negative number to represent higher or lower temperature than the previous day respectively. When the temperature was 14.8 °C on Wednesday, find the average temperature of the week.

<table>
<thead>
<tr>
<th>Day</th>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference °(C)</td>
<td>+1.2</td>
<td>+1.4</td>
<td>-0.8</td>
<td>+0.6</td>
<td>+2.2</td>
<td>-1.8</td>
<td></td>
</tr>
</tbody>
</table>

□1) The table below shows the sales figures of a product, using a positive or negative number to represent more or fewer pieces sold than the previous day respectively. Answer the following questions.

<table>
<thead>
<tr>
<th>Day</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference °(pieces)</td>
<td>-100</td>
<td>+1000</td>
<td>+200</td>
<td>-400</td>
<td>+300</td>
<td></td>
</tr>
</tbody>
</table>

□1) When 2400 pieces were sold on Monday, how many were sold on Wednesday?
□2) When 2400 pieces were sold on Friday, how many were sold on Monday?
□3) On which day did the product sell best?
1. Answer the following questions.

(1) In a card game, A has three 4-point cards, five 1-point cards, and four −2-point cards. How many points does A have?

(2) Four people, A, B, C, and D, are playing a game. The sum of their points is 2. When the average points of B, C, and D is −2, how many points does A have?

2. Fill in the blanks of the table on the right so that the sum becomes equal when adding any line of three numbers horizontally, vertically, or diagonally.

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>−1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>−4</td>
</tr>
</tbody>
</table>

3. The table below shows the difference between C’s weight and that of A to G, using kg as a unit. When the seven students’ average weight is 53.4 kg, answer the following questions.

<table>
<thead>
<tr>
<th>Student</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference* (kg)</td>
<td>−1.6</td>
<td>+2.7</td>
<td>0</td>
<td>+8.1</td>
<td>−3.2</td>
<td>+12.4</td>
<td>−7.9</td>
</tr>
</tbody>
</table>

* (Difference in weight compared to C(kg))

(1) How many kg lighter is G than B?

(2) Find E’s weight.

4. The table below shows the output of a product, using a positive or negative number to represent more or fewer pieces than the previous day respectively. When Day 3’s output was 100, answer the following questions.

<table>
<thead>
<tr>
<th>Day</th>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
<th>Day 5</th>
<th>Day 6</th>
<th>Day 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference* (pieces)</td>
<td></td>
<td>+21</td>
<td>−8</td>
<td>−10</td>
<td>+5</td>
<td>−24</td>
<td>+15</td>
</tr>
</tbody>
</table>

* (Difference in daily output compared to the previous day (pieces))

(1) How many pieces were produced on Day 5?

(2) If you use a positive number to show how many more the daily output was than Day 4’s, how do you express Day 7’s output?

(3) How many pieces is the average daily output of these seven days?
1. **The size of signed numbers**  Arrange the following numbers in ascending order of size.

- (1) 4, -2, -5  
- (2) 0, -0.1, 0.01  
- (3) $-\frac{1}{2}$, $-\frac{1}{3}$, -0.75

2. **Absolute values**  Answer the following questions.

- (1) List all the numbers whose absolute value is 16.
- (2) List all the integers whose absolute value is 2 or more and 7 or less in ascending order of size.
- (3) What is the greatest negative integer whose absolute value is greater than $\frac{7}{3}$?

3. **Addition and subtraction with signed numbers**  Calculate.

- (1) $(-3) - (+6)$  
- (2) $4 + (-9)$  
- (3) $4 - (-3) + (-2)$

- (4) $1.7 - 3.2 + 0.9$  
- (5) $-7 + 8 - 6 - 12$  
- (6) $-\frac{3}{8} - \left(-\frac{2}{5}\right)$

- (7) $-\frac{3}{4} + \frac{1}{2} - \left(-\frac{2}{3}\right)$  
- (8) $-\frac{1}{6} + \frac{2}{15} - \left(-\frac{1}{3}\right) - \frac{9}{10}$

4. **Multiplication and division with signed numbers**  Calculate.

- (1) $(-12) \times (-3)$  
- (2) $(-8) \div 2$  
- (3) $(-4)^2$

- (4) $(-3)^2 \times (-2^2) \times 5$  
- (5) $\frac{1}{3} \times \frac{4}{5} \div \left(-\frac{2}{3}\right)$  
- (6) $\left(-\frac{3}{4}\right)^2 \times \frac{2}{3} \div \left(-\frac{1}{2}\right)^2$

5. **Calculations with the four arithmetic calculations**  Calculate.

- (1) $16 - (-12) \times (-2)$  
- (2) $5 + (-3)^2 - 7$  
- (3) $(-2)^3 + 3 \times (-4)$

- (4) $5 + (1 - 4) \times 2$  
- (5) $-8 - 6 \times (4 - 7)$  
- (6) $12 \div 3 \times 2 - 15$

- (7) $0.6 + 0.4 \times (-7)$  
- (8) $(2.8 - 6.58) \div 0.7$

- (9) $\frac{5}{8} - \frac{1}{6} \div \frac{2}{9}$  
- (10) $\frac{1}{2} \times \left(-\frac{1}{3}\right) - \frac{1}{2} \div 3$
There are three points, A, B, and C, on the number line. When point A represents -4, point B represents 0.8, and point C represents 3, answer the following questions.

1. Find the number that is -5 greater than the number represented by point B.

2. Find the number represented by the point in the middle of point A and point C.

3. Point A is in the middle of point B and point D. Find the number represented by point D.

2. Calculate.

   - (1) \((-15) \div 3 - (-3)^2 \times (-7)\)
   - (2) \((-2)^2 + 8 - 5^3 \times (-1)^3\)
   - (3) \(-3^3 \times 2 - (-4)^3 \div (-2)^4\)
   - (4) \(13 - (2)^3 - (6 - 9) \times 2^4\)
   - (5) \(\left(\frac{1}{2} - \frac{1}{3}\right) \div \left(\frac{3}{8} - 1\right)\)
   - (6) \(2 \div \frac{2}{3} - \left(-\frac{3}{4}\right) \times (-2)^4\)
   - (7) \(\frac{2}{3} \div \left(-\frac{3}{2}\right) \div \frac{1}{4} + \left(-\frac{5}{6}\right)\)
   - (8) \(\left\{\frac{1}{2} \div 0.25 - \left(-\frac{3}{4}\right)^2\right\} \times \left(1 - \frac{7}{23}\right)\)

3. A and B tossed a coin seven times each, on condition that they got 3 points for heads and -5 points for tails. When the sum of their scores is -6, answer the following questions.

   - (1) How many heads and how many tails did A get, and how many heads and how many tails did B get?
   - (2) When A’s score is 16 points higher than B, how many tails did B get?

4. Answer the following questions.

   - (1) If \(a > 0\) and \(b < 0\), find all of the following expressions that always result in positive numbers and answer using symbols (i) to (v).
     (i) \(a - b\)  (ii) \(b - a\)  (iii) \(a \times b\)  (iv) \(-(a \div b)\)  (v) \(a^2 + b^2\)

   - (2) If \(-1 < a < 0\), arrange \(a\), \(-a\), \(a^2\), \(-a^2\) in ascending order of size.
1 How to express products
(1) Expressions with multiplication containing letters are written without using × symbols. Numbers are always written in front of letters, and letters are usually arranged in alphabetic order.
(2) \( a \times 1 \) and \( 1 \times a \) are written as \( a \), instead of \( 1a \). \((-1) \times a \) is written as \(-a\).
(3) The product of a letter and itself is written using exponents of powers: \( a \times a \times a = a^3 \)

2 How to express quotients
(1) Expressions with division containing letters are written in fraction form without using ÷ symbols.
(2) \( \frac{a}{2} \) can also be written as \( \frac{1}{2}a \): \( a \div 2 - a \times \frac{1}{2} = \frac{1}{2}a = \frac{a}{2} \)
(3) When the numerator or the denominator has a negative sign, it is put in front of the fraction: \( \frac{-3}{x} = \frac{-3}{x} \)

3 How to express quantities
When you write an expression that uses letters to indicate quantities, you should first explain by words and then replace them by numbers and letters.

4 Substitution and the value of expressions
(1) Plugging in a number for a letter in an expression is called substitution.
(2) The result of calculation by substituting a number for a letter is called the value of expression.

5 Terms and coefficients. Combining terms
(1) In an expression in addition form, each item joined by + signs is called a term.
(2) In a term using a letter, the number in front of the letter is called the coefficient of the letter.
(3) Terms with the same letter portion (or "like terms") can be simplified by combining them into one term.

6 Linear expressions
\( 3x - 5 \) consists of two terms: \( 3x \) and \( 5 \). Terms like \( 3x \) that contain only one letter are called terms of the first degree. An expression that contains only terms of the first degree or that indicates the sum of terms of the first degree and a number is called a linear expression.

7 Calculation with linear expressions
(1) To add or subtract with two linear expressions, first remove the parentheses, and then combine the terms having the same letter portion and those consisting of only numbers.
(2) To multiply a linear expression by a number, multiply each term of the expression by the number, using the distributive law as shown on the right.
(3) To divide a linear expression by a number, divide each term of the expression by the number. You can also multiply each term by the reciprocal of the divisor.

8 How to express the relations of quantities
(1) An expression that uses an equality sign (=) to indicate an equivalent relationship between two quantities is called an equality.
(2) An expression that uses an inequality sign (\(<\), \(\ge\)) to indicate the relative size of two quantities is called an inequality.
Let's learn the basics  1  Expressions that use letters

**Question**  Answer the following questions.

1. Write an expression to indicate the payment for three 80-yen goldfish and a 700-yen fishbowl to put them in.
2. Write an expression to indicate the payment for a 80-yen goldfish and a 700-yen fishbowl to put them in.

**Solution**

1. The expression to indicate the payment is \((\text{the price of each goldfish}) \times (\text{the number of goldfish}) + (\text{the price of a fishbowl})\), so \(80 \times 3 + 700\).
2. Even if the number of fish is not 3, for example 2 or 5, the expression to indicate the payment is still \(80 \times (\text{the number of goldfish}) + 700\).

Thus, the payment for \(a\) 80-yen goldfish and a 700-yen fishbowl to put them in is \(80 \times a + 700\) yen.

This expression indicates the payment when the number of goldfish is \(a\).

**Answer**

1. \(80 \times 3 + 700\) yen  
2. \(80 \times a + 700\) yen

---

1. You are going to buy some 150-yen apples with a 200-yen basket to put them in. Answer the following questions.

   □ (1) Fill in the blank of the verbal expression below.
   \((\text{the price of an apple}) \times (\text{[blank]}) + (\text{the price of the basket})\)

   □ (2) Write an expression that indicates the payment for \(n\) 150-yen apples and a 200-yen basket to put them in.

2. Write an expression to indicate the following quantities.

   □ (1) the circumference of a square whose sides are \(a\) cm

   □ (2) the change given when you pay 1000 yen for \(x\)-yen shopping

   □ (3) the travel distance you walk in \(x\) hours at a speed of 3 km per hour

   □ (4) the time it takes to walk 20 km at a speed of \(x\) km per hour

   □ (5) the price of a 10-m ribbon that cost \(a\) yen a meter

   □ (6) the payment for \(a\) 50-yen tangerines and \(b\)-yen apples

---

**Point**

1. When writing an expression that contains letters, first explain by words and then replace the words with letters.
### Chapter 2: Algebraic Expressions

#### Let's learn the basics

Remove $\times$ symbols, write numbers in front of letters, and write letters in alphabetic order.

1. $b \times 6 \times a = 6ab$
   - Remove $\times$ and write the number in front of the letters arranged in alphabetic order.

2. $(x-5) \times 2$
   - Write 2 in front of $(x)$. Write $(x)$ instead of $x$. $(-1) \times a$ is written as $-a$.

3. $x \times x \times x = x^3$
   - The product of a letter and itself is written using exponents of powers.

4. $(a-b) \times (a-b)$
   - The product of an expression enclosed with $( )$ is also written using exponents of powers.

(Note) $a \times 1$ and $1 \times a$ are written as $a$, instead of $1a$. $(-1) \times a$ is written as $-a$.

#### 3 Write the following expressions without using $\times$ symbols.

- (1) $a \times x$
- (2) $y \times b$
- (3) $a \times 9$
- (4) $y \times 4 \times a$
- (5) $x \times b \times 1$
- (6) $-3 \times b$
- (7) $y \times (-6)$
- (8) $x \times (-5) \times y$
- (9) $x \times 0.2 \times a$
- (10) $a \times b \times 7 \times c$
- (11) $x \times a \times b \times (-1)$
- (12) $a \times b \times (-0.1) \times m$

#### 4 Write the following expressions without using $\times$ symbols.

- (1) $7 \times (x-y)$
- (2) $(a-b) \times 12$
- (3) $(x-5) \times (-8)$
- (4) $a \times (a-b)$
- (5) $(a+b-c) \times 9$
- (6) $a \times (x+y) \times 2$
- (7) $b \times (-4) \times (b-1)$
- (8) $(a-b) \times (x+y)$

#### 5 Write the following expressions without using $\times$ symbols.

- (1) $a \times a$
- (2) $b \times b \times b$
- (3) $a \times b \times b$
- (4) $y \times y \times x \times (-7)$
- (5) $x \times b \times b \times x \times x$
- (6) $m \times n \times m \times n \times m$
- (7) $(a+b) \times (a+b) \times (a+b)$
- (8) $(x+2) \times (x+2) \times (x+2) \times (x+2)$
- (9) $(m+n) \times (m+n) \times (-5)$
- (10) $(a+3) \times (b-5) \times (a+3)$

#### 6 Write the following expressions using $\times$ symbols.

- (1) $5x$
- (2) $3ax$
- (3) $2(x-y)$
- (4) $6y^2$
- (5) $-7x^2y$
- (6) $(x+5) \times (y-6)$
- (7) $4(a-b)^2$
- (8) $3m(n+5)^2$
1 How to write algebraic expressions

Let's learn the basics

Expressions with division containing letters are written in fraction form without ÷ symbols.

\[(1) \frac{a}{5} \quad (2) \frac{p}{-q} \quad (3) \frac{x-y}{a}\]

- **Write the negative sign in front of the fraction.**
- **Remove the parentheses.**

[Note] An expression like $\frac{a}{2}$ can also be written as $\frac{1}{2}a$, $\frac{2x}{3}$ as $\frac{2}{3}x$, and $\frac{a+b}{2}$ as $\frac{1}{2}(a+b)$.

7 Write the following expressions without using ÷ symbols.

- (1) $x \div 9$
- (2) $a \div 6$
- (3) $y \div (-3)$
- (4) $a \div b$
- (5) $p \div q$
- (6) $x \div (-y)$
- (7) $x^2 \div 4$
- (8) $2 \div 5y$
- (9) $(-5b) \div 8$
- (10) $4x \div (-7)$
- (11) $m \div 2n$
- (12) $-a \div 5b$

8 Write the following expressions without using ÷ symbols.

- (1) $(x+y) \div 2$
- (2) $(a-2) \div 6$
- (3) $(a+b) \div (-3)$
- (4) $(p-q) \div r$
- (5) $(3x+y) \div (-z)$
- (6) $(a-b) \div 4c$
- (7) $5 \div (x-3)$
- (8) $-a \div (b+8)$
- (9) $(x+6) \div (y-2)$

9 Write the following expressions using ÷ symbols.

- (1) $\frac{y}{x}$
- (2) $\frac{-b}{3}$
- (3) $\frac{5a}{9}$
- (4) $\frac{a-b}{9}$
- (5) $\frac{x+y}{z}$
- (6) $\frac{2}{x+1}$
## Let's learn the basics

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x \times 2$</td>
<td>$6x$</td>
<td>Multiply the product of numbers by the letter.</td>
</tr>
<tr>
<td>$\frac{3}{4} \times (-8b)$</td>
<td>$-6b$</td>
<td></td>
</tr>
<tr>
<td>$6a \div (-3)$</td>
<td>$-2a$</td>
<td>Cancel down the number portion of fraction to its lowest terms.</td>
</tr>
<tr>
<td>$3 \times 6a$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-3 \times 8$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-5 \times (-9m)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5 \times (-7a)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4x \times 0.1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10 \times (-0.4x)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6 \times \frac{1}{18}y$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$35b \times \left(-\frac{1}{5}\right)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{8}a \times (-24)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 10 Calculate.

- (1) $2x \times 5$
- (2) $3 \times 6a$
- (3) $(-3) \times 7b$
- (4) $4 \times (-2y)$
- (5) $-6x \times 8$
- (6) $-5 \times (-9m)$
- (7) $(-x) \times (-1)$
- (8) $5 \times (-7a)$
- (9) $4x \times 0.1$
- (10) $(-3x) \times (-0.2)$
- (11) $10 \times (-0.4x)$
- (12) $6 \times \frac{1}{18}y$
- (13) $-\frac{2}{3}x \times 9$
- (14) $35b \times \left(-\frac{1}{5}\right)$
- (15) $\frac{1}{8}a \times (-24)$

### 11 Calculate.

- (1) $2 \times 3x \times (-6)$
- (2) $7 \times (-4) \times (-2y)$
- (3) $0.4x \times 8 \times 5$
- (4) $15 \times (-2) \times (-0.1a)$
- (5) $5 \times \frac{1}{20}b \times 3$
- (6) $\left(-\frac{1}{3}\right) \times 0.6x \times 10$

### 12 Calculate.

- (1) $9x \div 3$
- (2) $(-8x) \div 2$
- (3) $(-20b) \div 5$
- (4) $36x \div (-4)$
- (5) $-7y \div (-7)$
- (6) $(-42x) \div (-6)$
- (7) $2a \div (-10)$
- (8) $8x \div 6$
- (9) $(-12a) \div 8$
- (10) $(-6y) \div (-24)$
- (11) $12b \div 15$
- (12) $10p \div (-25)$

### Point

1. Don't write $\frac{5}{4}a$ as $\frac{1}{4}a$. 

---

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How to write algebraic expressions

Let's learn the basics

1. Expressions with multiplication or division containing letters can be written without using $\times$ or $\div$ symbols.
2. When dividing a term by a number or a letter, you can multiply it by the reciprocal of the divisor.
3. $+$ signs and $-$ signs cannot be removed.

\[
\begin{align*}
(1) \quad x \div (a \times b) &= \frac{x}{ab} \\
(2) \quad a \div b \div c &= \frac{1}{b} \times \frac{1}{c} \\
(3) \quad a \times 4 + b \div 7 &= \frac{4a + b}{7}
\end{align*}
\]

[Note] $a \div b \div c = (a \div b) \div c$ is right, but $a \div b \div c$ cannot be expressed as $a \div (b \div c)$.

13. Write the following expressions without using $\times$ or $\div$ symbols.

\[
\begin{align*}
(1) \quad a \times 5 \div b \\
(2) \quad a \div 5 \times b \\
(3) \quad a \div 5 \div b \\
(4) \quad a \times 7 \div b \\
(5) \quad a \div (7 \times b) \\
(6) \quad a \div (7 \div b) \\
(7) \quad x \div yz \\
(8) \quad ax \div by \\
(9) \quad 2pq \div mn
\end{align*}
\]

14. Write the following expressions without using $\times$ or $\div$ symbols.

\[
\begin{align*}
(1) \quad a \times a \times 2 \div b \\
(2) \quad x \div (-3) \times x \div y \\
(3) \quad (a+b) \times (a+b) \div x \\
(4) \quad (a-b) \times 5 \div x \div x
\end{align*}
\]

15. Write the following expressions without using $\times$ or $\div$ symbols.

\[
\begin{align*}
(1) \quad x-y \times 3 \\
(2) \quad a \times 7 + b \div 2 \\
(3) \quad 9 \div x - y \div 7 \\
(4) \quad a \times a \times 6 - 8 \div b \div b \\
(5) \quad (x-y \times 4) \div (a+b) \\
(6) \quad -3 \times x \times x - 5y \times 2 \\
(7) \quad 4 \times a \div b \div b \times a + 6b \div (-2)
\end{align*}
\]

16. Write the following expressions using $\times$ and $\div$ symbols.

\[
\begin{align*}
(1) \quad \frac{ax}{3} \\
(2) \quad \frac{3x^2}{4} \\
(3) \quad \frac{b}{ax} - \frac{y^2}{4}
\end{align*}
\]
1. Write the following expressions without using × symbols.
- (1) \(a \times 5\)
- (2) \(x \times (-2) \times y\)
- (3) \(x \times (-0.01)\)
- (4) \((m+n) \times 7\)
- (5) \((a-b) \times (-2)\)
- (6) \(x \times (y+6) \times 4\)
- (7) \(a \times b \times a \times c\)
- (8) \(x \times y \times y \times (-1)\)
- (9) \((x-5) \times (x-5)\)

2. Write the following expressions without using ÷ symbols.
- (1) \(p \div 7\)
- (2) \(y \div (-4)\)
- (3) \((-2x) \div (-9)\)
- (4) \((x-3) \div 5\)
- (5) \(a \div (x+y)\)
- (6) \((x+y) \div (a-b)\)

3. Calculate.
- (1) \(3x \times 4\)
- (2) \(5 \times (-2b)\)
- (3) \((-8a) \times (-3)\)
- (4) \((-6) \times 0.2c\)
- (5) \(\frac{3}{32}x \times 8\)
- (6) \(4 \times (\frac{-1}{12}y) \times 2\)
- (7) \(15a \div (-3)\)
- (8) \(24x \div 20\)
- (9) \((-\frac{16}{a}) \div 8\)

4. Write the following expressions without using × or ÷ symbols.
- (1) \(4 \times x \div y\)
- (2) \(3 \div x \times y\)
- (3) \(8 \div (a \times b)\)
- (4) \(a \div 3 \times b \div c\)
- (5) \(p \times 3 + q \div 2\)
- (6) \(x \div 7 - 4 \div y\)
- (7) \(x \times x \div 2 + x \times 5\)
- (8) \((a-b \times 3) \div (x+10)\)
- (9) \(-6 \times a \times a \div b + a \div 8\)
- (10) \(8 \div x \div x \times y - 7 \div x \times y\)

5. Write the following expressions using × and ÷ symbols.
- (1) \(3ab^2\)
- (2) \(-\frac{18}{x}\)
- (3) \(\frac{y}{x}\)
- (4) \(\frac{2a}{5b}\)
- (5) \(\frac{(a-b)^2}{2}\)
- (6) \(\frac{5}{4x} - \frac{3a}{y}\)
Let's learn the basics

1 How to express costs and numbers

**Question** What is the total cost for \( a \) 120-yen apples and \( b \) 30-yen tangerines?

**Solution** The total cost is (the cost for apples) + (the cost for tangerines).

The cost of apples is \( 120 \times a \) (yen), and the cost of tangerines is \( 30 \times b \) (yen).

So, the total cost is \( 120a + 30b \) (yen).

**Answer** \( 120a + 30b \) (yen)  
(Note) This expression can also be written as \( (120a + 30b) \) yen.

\[ \text{Dividend} = (\text{Divisor}) \times \text{(Quotient)} + \text{(Remainder)} \]

- Multiples of 3 are expressed as \( 3 \times \) (Integers)
- Three consecutive integers can be expressed as \( n, n+1, n+2 \), \( n-1, n, n+1 \), etc. (\( n \) is an integer.)

1 Write an expression to indicate the following quantities.

- (1) the cost of \( x \) 80-yen pencils
- (2) the change given when you pay for \( x \) a-yen erasers with a 1000-yen bill
- (3) the total cost of \( m \) 120-yen bottles of juice and \( n \) 300-yen cakes
- (4) the amount of money left when five people contribute \( a \) yen each and buy \( b \) 30-yen tangerines

2 Write an expression to indicate the following quantities.

- (1) the number that leaves a quotient of \( a \) with a remainder of 5 when divided by 7
- (2) the number that leaves a quotient of 8 with a remainder of \( y \) when divided by \( x \)
- (3) express the following numbers by using \( n \) as an integer
  - multiples of 5
  - multiples of 8
  - even numbers
  - odd numbers
- (4) three consecutive integers when the smallest one is \( x \)
- (5) a double-figure natural number whose tens digit is \( x \) and ones digit is \( y \)

**Point**

1. When writing expressions that use letters to indicate quantities, you must follow the rules about the use of letters.
Chapter 2 Algebraic expressions

Let's learn the basics

**Question 1** There are three people whose heights are \(a\) cm, \(b\) cm, and \(c\) cm, respectively. How many centimeters is their average height?

**Solution** Since the average is (the sum of heights) ÷ (the number of people), \((a+b+c) ÷ 3\) (cm).

**Answer** \(\frac{a+b+c}{3}\) cm

**Question 2** Answer the following questions.

1. How many grams is the sum of \(a\) kg and \(b\) g?  
2. How many meters is the sum of \(x\) m and \(y\) cm?

**Solution** When writing an expression to indicate the sum or difference of two quantities with different units, use one of them as the common unit.

1. Since 1 kg = 1000 g, \(a\) kg can be expressed as 1000a g. Thus, 1000a + b (g).

2. Since 1 cm = \(\frac{1}{100}\) m, \(y\) cm can be expressed as \(\frac{y}{100}\) m. Thus, \(x + \frac{y}{100}\) (m).

**Answer** (1) 1000a + b (g)  
(2) \(x + \frac{y}{100}\) (m)

**3** Write an expression to indicate the following quantities.

- (1) the average score of four tests when the scores are \(a\), \(b\), \(c\), and \(d\) points
- (2) the amount of money that each of three people pays when they share the cost equally to buy 4 \(a\)-yen candies and 5 \(b\)-yen candies
- (3) the average of the total weight of 15 \(x\)-kg boys and 14 \(y\)-kg girls

**4** Express the following quantities using the unit in [ ].

- (1) \(x\) m [cm]  
- (2) \(y\) cm [mm]  
- (3) \(a\) mm [m]  
- (4) \(b\) km [m]

- (5) \(c\) g [kg]  
- (6) \(x\) m² [cm²]  
- (7) \(a\) hours [minute]  
- (8) \(b\) seconds [minute]

**5** Express the sum of the following quantities using the unit in [ ].

- (1) \(x\) cm and \(y\) mm [mm]  
- (2) \(x\) km and \(y\) m [km]

- (3) \(x\) kg and \(y\) g [kg]

- (4) \(a\) hours and \(b\) minutes [minute]  
- (5) \(p\) minutes and \(q\) seconds [minute]

- (6) \(x\) L and \(y\) dL [dL]
### How to express areas and volumes

#### Question 1

**How many cm² is the area of a triangle with a base of \(a\) cm and a height of \(h\) cm?**

**Solution**

Since \(\text{the area of a triangle} = \text{base} \times \text{height} \div 2\), \(a \times h \div 2 = \frac{1}{2} ah\) (cm²)

**Answer**

\( \frac{1}{2} ah\) cm²

#### Question 2

**How many centimeters is the circumference of a circle with a radius of \(r\) cm?**

**Solution**

When calculating the circumference or area of a circle, the ratio of the circumference of a circle to the diameter is expressed as \(\pi\) (pi).

Since \(\text{the circumference of a circle} = \text{diameter} \times \pi\) and the diameter is \(2r\) cm, \(2r \times \pi = 2\pi r\) (cm)

**Answer**

\(2\pi r\) cm

---

6. Fill in the blanks to complete the table below showing the areas and circumferences of the following figures. (Unit of the figures is cm, and use \(\pi\) for the ratio of the circumference of a circle to its diameter.)

<table>
<thead>
<tr>
<th></th>
<th>1 Triangle</th>
<th>2 Rectangle</th>
<th>3 Square</th>
<th>4 Rhombus</th>
<th>5 Trapezoid</th>
<th>6 Parallelogram</th>
<th>7 Circle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (cm²)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circumference (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. Write an expression to indicate the volume of the following solids.

- (1) a cube whose side is \(a\) cm
- (2) a cuboid \(a\) cm long, \(b\) cm wide, and \(c\) cm high

8. Write an expression using \(\pi\) to indicate the area and circumference of the following figures.

- (1) a circle whose radius is 8 cm
- (2) a circle whose radius is 5 cm
- (3) a semicircle whose radius is 4 cm
- (4) a semicircle whose radius is 3 cm
- (5) the area of the shaded part of the figure on the right (this quadrilateral is a square).

---

**Point**

1. The ratio of the circumference of a circle to its diameter is expressed as \(\pi\), which is the initial letter of a Greek word meaning “circumference.”
Let's learn the basics

4 How to express rates and speed

Question 1: There are $a\%$ of the 40 students in a class absent today. How many are absent?

Solution: Since $1\%$ is $\frac{1}{100}$, $a\%$ can be written as $\frac{a}{100}$ in fraction form. It can also be written as $0.01a$ in decimal form. Thus, $a\%$ of the 40 students are $40 \times \frac{a}{100} = \frac{2}{5}a$ (students) or $40 \times 0.01a = 0.4a$ (students).

Answer: $\frac{2}{5}a$ students (or $0.4a$ students)

Question 2: It took $y$ hours to go $x$ km by bicycle. How many kilometers per hour was the speed?

Solution: Since \((\text{speed}) = \frac{\text{(travel distance)}}{\text{(time)}}\), the speed is $\frac{x}{y}$ km/h. $\frac{x}{y}$ km/h means $\frac{x}{y}$ km per hour.

Answer: $\frac{x}{y}$ km/h

9 Answer the following questions.

☐ (1) How much is $p\%$ of 300 yen?

☐ (2) How many kilograms is $b$ wari of 500 kg?

☐ (3) How many is $6\%$ of $a$ people?

☐ (4) How much is $8$ wari of $x$ yen?

10 Write an expression to indicate the following quantities.

☐ (1) the tag price of an article when its cost price is 1000 yen and an $a\%$ profit is expected

☐ (2) the number of students who are present when $p\%$ of 600 students are absent

11 Express the following quantities using the unit in [ ].

☐ (1) the travel distance of walking for $b$ minutes at a speed of $a$ m per minute [m]

☐ (2) the time it takes to run 100 meters at a speed of $x$ m per second [second]

☐ (3) the meter per minute of going $x$ m in $y$ hours [m/min]

12 Answer the following questions.

☐ (1) How many grams of salt is contained in salt water whose concentration is 7%?

☐ (2) What percentage is the concentration of salt water made by mixing $a$ g of water and $b$ g of salt?

Point

1. \((\text{the amount of salt}) = (\text{the amount of salt water}) \times \frac{\text{the concentration of salt water}(\%)}{100}\)

2. \((\text{the concentration of salt water}) = \frac{(\text{the amount of salt})}{(\text{the amount of salt water})} \times 100(\%)\)
### Let's learn the basics  
**5. The value of expressions**

1. Plugging in a number for a letter in an expression is called **substitution**. The result of calculation by substituting a number for a letter is called the **value of the expression**.

2. When substituting a negative number, enclose it within parentheses.

<table>
<thead>
<tr>
<th>Question</th>
<th>Find the value of the following expressions when $a = -5$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $2a + 3$</td>
<td>(2) $3a^2 - a$</td>
</tr>
</tbody>
</table>

**Solution**

Substitute $-5$ for the letter $a$ in the expression.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Calculation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $2(-5) + 3$</td>
<td>$-10 + 3$</td>
<td>$-7$</td>
</tr>
<tr>
<td>(2) $3(-5)^2 - (-5)$</td>
<td>$75 + 5$</td>
<td>$80$</td>
</tr>
</tbody>
</table>

**Answer**

(1) $-7$  
(2) $80$

---

### Find the value of the following expressions when $a = 5$.  

1. $7a$  
2. $3a + 4$  
3. $8 - 2a$  
4. $\frac{2a}{3}$  
5. $a^2$  
6. $\frac{25}{a}$

---

### Find the value of the following expressions when $x = -4$.  

1. $2x + 6$  
2. $-5x + 13$  
3. $\frac{1}{2}x - 6$  
4. $-3x^2$  
5. $x^3$  
6. $2x^2 - 5x$

---

### Find the value of $p^2 - 2p + 3$ when the value of $p$ is the following.  

1. $p = 6$  
2. $p = -8$  
3. $p = -\frac{1}{2}$

---

### Find the value of the following expressions when $x = -5$ and $y = -4$.  

1. $2x + 3xy$  
2. $-4xy - 2y^2$  
3. $(x + 3y)^2$

---

### When you are going to sell an $x$-yen article at a 20% discount, answer the following questions.  

1. Write an expression using $x$ to indicate the selling price.  
2. How much is the selling price when $x = 2500$?
## Exercises

### 1. How to express quantities. The value of expressions

1. Write an expression to indicate the following quantities.
   - (1) the total cost for \(a\) 90-yen candies and \(b\) 150-yen cakes
   - (2) the average weight of \(3x\)-kg packages and \(2y\)-kg ones
   - (3) a three-figure natural number whose hundreds, tens, and ones digits are \(a\), \(b\), and \(c\), respectively

2. Write an expression to indicate the following quantities. Use \(\pi\) for the ratio of the circumference of a circle to its diameter.
   - (1) the weight of sugar you have left after using \(b\) g of \(a\) kg of sugar [Unit: g]
   - (2) the area of a right triangle in which \(p\)-cm side and \(q\)-cm side forms the right angle
   - (3) the area of a semicircle whose radius is \(r\) cm

3. Write an expression to indicate the following quantities.
   - (1) the payment for an \(x\)-yen article at a 25% discount
   - (2) the travel distance of running for \(y\) minutes at a speed of \(x\) km per hour [Unit: km]
   - (3) the concentration of salt water made by mixing 200 g of \(a\)% salt water and 100 g of water

4. Answer the following questions.
   - (1) Find the value of \(a^2+5a-7\) when \(a = -4\).
   - (2) Find the value of \(4x-3y\) when \(x = 3\) and \(y = -5\).
   - (3) Find the value of \(2a^2+b^2-6c\) when \(a = -3\), \(b = 4\), and \(c = 2\).

5. There is a cuboid that is \(a\) cm long, \(b\) cm wide, and \(b\) cm high. What does the following expression indicate about this cuboid? Write using a suitable unit.
   - (1) \(a \times b^2\)
   - (2) \(4a + 8b\)
Calculating linear expressions

Let's learn the basics

1 Terms and coefficients

In an expression in addition form, each item joined by + signs is called a term. In a term using a letter, the number in front of the letter is called the coefficient of the letter.

(1) About $5x - 3$,
since $5x - 3 = 5x + (-3)$,
the terms of this expression are $5x$ and $-3$,
and the coefficient of $x$ is $5$.

(2) About $4a^2 - a + 6$,
since $4a^2 - a + 6 = 4a^2 + (-a) + 6$,
the terms of this expression are $4a^2$, $-a$, and $6$,
and the coefficients of $a^2$ and $a$ are $4$ and $-1$, respectively.

1 Find the terms of the following expressions and the coefficient of the letters.

\( \begin{align*}
(1) & \quad 7a - 5 \\
(2) & \quad 8x + xy \\
(3) & \quad 5a^2 + 2ab - \frac{b^2}{3}
\end{align*} \)

2 Mark a circle (\(\bigcirc\)) if the following expression is a linear expression, and mark a cross (\(\times\)) if it is not.

\( \begin{align*}
(1) & \quad 3a \\
(2) & \quad x^2 \\
(3) & \quad 6y + 5 \\
(4) & \quad 2a^2 - a \\
(5) & \quad 7 - 12x \\
(6) & \quad \frac{3}{5}x + \frac{1}{2}y
\end{align*} \)

Let's learn the basics

2 Combining terms

Terms with the same letter portion (like terms) can be simplified by combining them into one term.

\( \begin{align*}
(1) & \quad 5x + 2x = (5+2)x = 7x \\
(2) & \quad -6y + 3y = (-6+3)y = -3y \\
(3) & \quad 3a - 8a = (3-8)a = -5a
\end{align*} \)

3 Calculate.

\( \begin{align*}
(1) & \quad 7x + 5x \\
(2) & \quad y + 3y \\
(3) & \quad 13a - a \\
(4) & \quad 2a - 7a \\
(5) & \quad -9x + 6x \\
(6) & \quad 8y + (-y) \\
(7) & \quad 15x - 8x \\
(8) & \quad -13y + 12y \\
(9) & \quad a - 10a \\
(10) & \quad -x - 9x \\
(11) & \quad -7a + 7a \\
(12) & \quad 11y - (-5y)
\end{align*} \)

Point

1 Terms that contain only one letter are called the terms of the first degree. An expression that contains only terms of the first degree or that indicates the sum of terms of the first degree and a number is called a linear expression.
Chapter 2  Algebraic expressions

3  Combining terms (2)

Let's learn the basics

To simplify an expression that contains some terms, combine terms with the same letter portion, and then do the same to those that are just numbers. When combining terms with the same letter portion, write the sum of the coefficients in front of the letter.

(1) \(4x + 5x - 12x\)

\[= (4 + 5 - 12)x\]
\[= -3x\]

(2) \(3y - 7 + 4y + 3\)

\[= 3y + 4y - 7 + 3\]
\[= (3 + 4)y + (-7 + 3)\]
\[= 7y - 4\]

Cannot be simplified anymore.

(Note) An expression that contains some coefficients in decimal or fraction form can also be simplified in the same way.

4  Calculate.

\[\text{(1)} \quad 9x - 2x - 5x\]

\[\text{(2)} \quad 8a - 12a + 3a\]

\[\text{(3)} \quad x - 3x + 5x\]

\[\text{(4)} \quad 7y - y - 10y\]

5  Calculate.

\[\text{(1)} \quad 3a + 4 + 5a + 2\]

\[\text{(2)} \quad 2x - 7 - 4x + 9\]

\[\text{(3)} \quad 2x - 3 - x + 8\]

\[\text{(4)} \quad -6y - 8 + 6y + 2\]

\[\text{(5)} \quad 25a - 23 - 13a + 18\]

\[\text{(6)} \quad 78 - 36x - 29 + 53x\]

\[\text{(7)} \quad 6 + 2b - 3 + 5b + 4\]

\[\text{(8)} \quad 15y - 4 - 7y - 8 + 3y\]

\[\text{(9)} \quad 26m - 14m - 36 - 15m + 38\]

\[\text{(10)} \quad 13 - 2p + 21 + 15p - 10 - 9p\]

6  Calculate.

\[\text{(1)} \quad 0.4a + 0.5 + 0.8a + 0.3\]

\[\text{(2)} \quad 0.2x - 0.6 - 0.3x + 0.8\]

\[\text{(3)} \quad -y + 3.7 + 2.8y - 5.2\]

\[\text{(4)} \quad \frac{3}{8}a - 9 - \frac{5}{8}a + 2\]

\[\text{(5)} \quad \frac{1}{2}x + \frac{1}{5} - \frac{4}{3}x - \frac{6}{5}\]

\[\text{(6)} \quad -\frac{1}{6} - \frac{2}{3}m + \frac{1}{3} + \frac{1}{4}m\]
Let's learn the basics

1. When adding two linear expressions, remove parentheses and simplify the terms by combining them.
2. When subtracting a linear expression from another, change the sign of each term of the expression to be subtracted and then add them.

(1) \((3x+5) + (4x-9)\)
\[= 3x + 5 + 4x - 9\]
\[= 7x - 4\]

(2) \((7a - 3) - (2a - 5)\)
\[= 7a - 3 - 2a + 5\]
\[= 5a + 2\]

7) Calculate.

1. \((2a + 3) + (3a + 4)\)
2. \((5a + 8) + (2a - 3)\)

3. \((7x - 5) + (2x + 8)\)
4. \((-2y + 3) + (5y - 9)\)

5. \((4 + 3a) + (3 - 5a)\)
6. \((-12x + 16) + (12x - 5)\)

8) Calculate.

1. \((8a + 3) - (5a + 1)\)
2. \((6a + 4) - (3a - 2)\)

3. \((7x - 3) - (-3x - 6)\)
4. \((2b - 6) - (-9b + 3)\)

5. \((5y - 11) - (5y - 18)\)
6. \((-13x + 17) - (13x - 9)\)

7. \(9x - (3 - 6x)\)
8. \((12y - 5) - (15y + 8)\)

9) Calculate.

1. \((0.5a + 0.4) + (0.3a - 0.2)\)
2. \((1.5x - 0.6) - (0.8x - 1)\)

3. \((-x - 2.7) - (-1.9x - 1.7)\)
4. \(\left(\frac{1}{5}x + 5\right) + \left(\frac{2}{3}x - 7\right)\)

5. \(\left(\frac{1}{3}a + 3\right) - \left(\frac{1}{2}a - 7\right)\)
6. \(\left(\frac{4}{9}y + \frac{2}{3}\right) - \left(\frac{1}{4} + \frac{5}{6}y\right)\)

10) Calculate.

1. \(-7x - 15 + x + 6\)
2. \(5a + 4 - 2a - 9\)
3. \(-4x - 6 - 3x - 7\)
Chapter 2  Algebraic expressions

Let's learn the basics  Multiplication and division with linear expressions and numbers

1. When multiplying a linear expression by a number, multiply each term in the expression by the number, using the distributive law.

2. When dividing a linear expression by a number, divide each term in the expression by the number or multiply them by the reciprocal of the divisor.

(1) \(2(3a+1)\) 
\[= 2 \times 3a + 2 \times 1\] 
\[= 6a + 2\] 

(2) \(\frac{2a-5}{3} \times 6\) 
\[= \frac{(2a-5) \times 6}{3}\] 
\[= (2a-5) \times 2\] 
\[= 4a - 10\] 

(3) \(\frac{12x+8}{4} \div 4\) 
\[= \frac{12x+8}{4} \times \frac{1}{4}\] 
\[= 3x + 2\]

11 Calculate.

1. \(3(2a+5)\) 
2. \(4(3a-2)\) 
3. \(6(4a+3)\) 
4. \(-(2x+3)\) 
5. \(-3(3x-4)\) 
6. \((a+4) \times 4\) 
7. \((5x+2) \times (-6)\) 
8. \((-y-7) \times 9\) 
9. \((-5m+8) \times (-3)\)

12 Calculate.

1. \(\frac{3}{4}(8a-12)\) 
2. \(-\frac{2}{3}(6a-1)\) 
3. \((2a-10) \times \frac{1}{2}\) 
4. \((28a-7) \times \left(-\frac{3}{14}\right)\) 
5. \(12 \left(\frac{3}{4}x-\frac{5}{6}\right)\) 
6. \(\left(-\frac{5}{6}x-\frac{1}{4}\right) \times (-24)\) 
7. \(4 \times \frac{x-1}{2}\) 
8. \(6 \times \frac{2x+3}{2}\) 
9. \(-14 \times \frac{x-5}{7}\) 
10. \(\frac{3x+1}{2} \times 4\) 
11. \(\frac{5x-4}{3} \times 12\) 
12. \(\frac{2y+5}{3} \times (-15)\)

13 Calculate.

1. \((6a+15) \div 3\) 
2. \((24x-12) \div (-4)\) 
3. \((4x-10) \div 6\) 
4. \(\left(\frac{2}{3}a+8\right) \div (-2)\) 
5. \(\left(\frac{3}{8}x-9\right) \div \left(-\frac{1}{3}\right)\) 
6. \(\left(\frac{7}{12}x+\frac{3}{4}\right) \div \frac{7}{24}\)
3 Calculating linear expressions

Let's learn the basics

You can remove the parentheses in expressions and combine the terms using the distributive law. Don't forget to change the sign of each term when multiplying an expression by a negative number.

(1) \[2(3x+4)+3(x-1)\]
Remove the parentheses.
\[=6x+8+3x-3\]
\[=9x+5\]

(2) \[6(a-2)-4(2a-5)\]
\[=6a-12-8a+20\]
\[=-2a+8\]

Various calculations

You can use either of the following methods.

1. Change the two fractions to those with a common denominator.
2. The distributive law

14 Calculate.

- (1) \[4a+2(3a-1)\]
- (2) \[5a-3(a-2)\]
- (3) \[5x+3+2(1-x)\]
- (4) \[2(x+1) - (x-3)\]
- (5) \[3(2a+1)+2(a+2)\]
- (6) \[4(2x+5)+3(x+2)\]
- (7) \[5(3y+4)-2(4y-1)\]
- (8) \[2(3x-6)+6(-x+3)\]
- (9) \[-4(2a+7)-5(-3a-6)\]
- (10) \[7(x-4)-2(3x-9)\]
- (11) \[-4(a+2)+3(2a-1)\]
- (12) \[7(6x-5)-2(3x-8)\]

15 Calculate.

- (1) \[6\left(a+\frac{1}{3}\right)+8\left(2a-\frac{1}{2}\right)\]
- (2) \[4\left(\frac{1}{2}x+1\right)-9\left(\frac{2}{3}x-2\right)\]
- (3) \[\frac{1}{4}(12x-16)-\frac{1}{5}(30x-50)\]
- (4) \[\frac{1}{3}(6x+9)+\frac{3}{4}(4x+8)\]
- (5) \[\frac{3x+1}{2}\times8+\frac{x-2}{3}\times9\]
- (6) \[14\times\frac{a-5}{2}-12\times\frac{2a-1}{3}\]

16 Calculate.

- (1) \[\frac{a-2}{3}+\frac{a+1}{2}\]
- (2) \[\frac{a-1}{2}-\frac{a-1}{4}\]
- (3) \[\frac{3x+2}{2}-\frac{x-2}{3}\]
- (4) \[\frac{4x-5}{14}-\frac{3x-2}{7}\]
- (5) \[a-\frac{3a-1}{5}\]
- (6) \[\frac{2a-1}{3}+2a\]
- (7) \[\frac{2x+3}{5}\times\frac{x+4}{10}\]
- (8) \[\frac{2(x+4)}{3}+\frac{x-3}{5}\]
- (9) \[\frac{4(x-3)}{3}-\frac{5(x-1)}{2}\]
Calculate.

□(1) $7x - 6x - 9x$

□(2) $13x - 8 - 15x + 24$

□(3) $-0.29x + 3 + x - 1.7$

□(4) $\frac{2}{3}y - \frac{1}{4} - \frac{5}{6}y - \frac{1}{2}$

□(5) $(6x - 9) + (4x + 13)$

□(6) $(8x + 4) - (3x - 2)$

□(7) $(0.9a - 15) + (-0.1a + 8)$

□(8) $\left(\frac{3}{4}y - \frac{1}{2}\right) - \left(\frac{5}{6}y - \frac{2}{3}\right)$

Calculate.

□(1) $8(4x + 6)$

□(2) $-5(2x - 1)$

□(3) $\frac{3a - 5}{8} \times 56$

□(4) $(30a - 12) \div 6$

□(5) $(15a - 10) \div (-5)$

□(6) $\left(\frac{8}{11}x + \frac{24}{25}\right) \div 12$

Calculate.

□(1) $2(5a + 3) + 4(2a - 1)$

□(2) $7(3x - 5) - 9(x - 6)$

□(3) $3\left(\frac{1}{3}a + \frac{1}{6}\right) - 4\left(\frac{1}{2}a - \frac{5}{12}\right)$

□(4) $\frac{2x - 1}{3} - \frac{3x - 3}{4}$

□(5) $\frac{1}{3}(3x - 7) - \frac{2}{3}(x - 1)$

□(6) $2(2x + 3) + 4(x - 5) - 3(3x - 6)$

Fill in [ ] with a suitable expression.

□(1) $(3x - 5) + (\square) = x - 8$

□(2) $(2x + 7) - (\square) = 5x + 6$

□(3) $2(x - 6) + 3(\square) = 8x + 3$

□(4) $5(4x - 3) - 4(\square) = 12x + 5$

Write the following expressions using $x$ when $A = 2x + 1$ and $B = 3x - 2$.

□(1) $A + B$

□(2) $A - B$

□(3) $2A + 3B$
### 4 Using algebraic expressions

#### Let's learn the basics

**1 Applying calculations with algebraic expressions**

<table>
<thead>
<tr>
<th>Question</th>
<th>The fee for adults to enter a botanical garden is (a) yen, and that for children is 120 yen lower. What is the total fee for 2 adults and 3 children to enter the botanical garden?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solution</strong></td>
<td>The entrance fee for 2 adults is (a \times 2 = 2a) (yen).&lt;br&gt;The entrance fee for a child is ((a - 120)) yen, so the fee for 3 children is (3(a - 120)) yen. &lt;br&gt;Thus, the total fee to enter the botanical garden is (2a + 3(a - 120) = 2a + 3a - 360 = 5a - 360) (yen).</td>
</tr>
<tr>
<td><strong>Answer</strong></td>
<td>(5a - 360) (yen)</td>
</tr>
</tbody>
</table>

1. Answer the following questions.

   1. The fee for adults to enter a zoo is \(a\) yen, and that for children is 150 yen lower. Write an expression to indicate the total fee for 5 adults and 8 children to enter the zoo.

   2. A has \(a\) yen. B has 100 yen more than a half amount of A's money, and C has 50 yen less than twice the amount of A's money. Write an expression to indicate the total sum of money these three people have.

   3. You bought 20 candies, which were either 60 or 80 yen apiece. When you got \(x\) 60-yen candies, how much did you pay?

   4. You walked 12 km, first at a speed of 3 km per hour, and then at a speed of 4 km per hour. When you walked \(x\) km at a speed of 3 km per hour, how long did it take to walk the distance?

2. There is a rectangle 6 cm long and 9 cm wide. When you double the length and make the width \(x\) cm shorter, answer the following questions about the new rectangle.

   1. What is the circumference?  
   2. What is the area?

3. What is the circumference and area of the shaded part of the following figure? Use \(\pi\) for the ratio of the circumference of a circle to its diameter.

   1.  
   2.  
   3. 

### 5 Point

1. Write an algebraic expression to indicate quantities and make it as simple as possible by calculation.
As shown in the figure on the right, you can make equilateral triangles by arranging matchsticks from left to right. Answer the following questions.

(1) How many matchsticks do you need to make 5 equilateral triangles?
(2) How many matchsticks do you need to make \( n \) equilateral triangles?

**Solution**

(1) Dividing the arranged matchsticks into the parts shown below, you find that the number of matchsticks you need to make 5 equilateral triangles is 

\[
1 + 2 \times 5 = 11
\]

(2) Starting with one matchstick, you can make one equilateral triangle by adding two matchsticks each time. So the number of matchsticks you need to make \( n \) equilateral triangles is 

\[
1 + 2 \times n = 2n + 1
\]

**Answer**

(1) 11 matchsticks  
(2) \( 2n + 1 \) (matchsticks)

---

As shown in the figure on the right, you can make regular hexagons by arranging matchsticks from left to right. Answer the following questions.

(1) How many matchsticks do you need to make 5 regular hexagons?
(2) How many matchsticks do you need to make \( n \) regular hexagons?

---

As shown in the figure on the right, you are sticking pieces of tape 8 cm long together in a line from left to right with 1 cm overlapping each other. Answer the following questions.

(1) How long are 10 pieces of tape stuck together?
(2) How long are \( n \) pieces of tape stuck together?

---

As shown in the figure on the right, you are putting squares with a side of 2 cm together so that the middle of one side of a square meets a vertex of another square. Answer the following questions.

(1) What is the circumference of the figure made by putting 6 squares together?
(2) What is the circumference of the figure made by putting \( n \) squares together?
(3) What is the circumference of the figure made by putting 25 squares together?
Let's learn the basics  How to express the relationships between quantities

1. Equalities

As shown on the right, an expression that uses the equal sign (=) to indicate a relationship between two quantities is called an equality.

The expression to the left of the equal sign is called the left side of an equality, and the one to the right side is called the right side of it. They are collectively called both sides of the equality.

Example: An equivalent relationship between two quantities can be expressed using an equal sign as shown below.

(1) Three times a number $x$ plus 2 is equal to four times the difference $x$ minus 5

$$3x + 2 = 4(x - 5)$$

(2) You had $y$ pencils. You distributed them to $x$ children so that each of them got 3 pencils, and you have nine pencils left now.

$$y = 3x + 9$$

7. Express the relationships between the following quantities using an equality.

☐1. Five times a number $x$ is equal to three times a number $y$ plus 6.

☐2. A number $a$ divided by 8 is $b$ with a remainder of 5.

☐3. Three times the difference of a number $x$ minus 2 is equal to $x$ plus 4.

8. Express the relationships between the following quantities using an equality.

☐1. You want to distribute $y$ tangerines to $x$ children so that each of them gets 4 tangerines, but you are 6 tangerines short.

☐2. There is 2000 mL of juice. If 6 people drink $x$ mL each, $y$ mL is left.

☐3. It cost 1250 yen to buy 5 $x$-yen apples and 3 $y$-yen peaches.

9. Express the relationships between the following quantities using an equality.

☐1. You paid 5000 yen for an $x$-yen article at 2 wari off, and got the change of $y$ yen.

☐2. You traveled $y$ km by walking for two hours and a half at a speed of $x$ km an hour.
Let's learn the basics 4 How to express the relationships between quantities

(2) Inequalities

As shown on the right, an expression that uses the inequality sign

\( <, >, \leq, \geq \)

to indicate a relationship of two quantities is called an inequality.

Like an equality, the expressions to the left and right of the inequality sign are called the left side and right side, respectively, and they are collectively called both sides of the inequality.

The signs \( > \) and \( < \) mean “more than” and “less than,” respectively, while the signs \( \geq \) and \( \leq \) represent “at least” and “at most,” respectively.

Example  The relative size between two quantities can be expressed using an inequality sign as shown below.

(1) A number minus 4 is more than 5.

\[ x - 4 > 5 \]

(2) A boy and his sister weigh \( a \) kg and \( b \) kg, respectively. Their average weight is less than 50 kg.

\[ \frac{a + b}{2} < 50 \]

(3) The total cost of 2 \( a \)-yen notebooks and 3 \( b \)-yen pens is at least 500 yen.

\[ 2a + 3b \geq 500 \]

(4) It takes at most an hour to go to a place \( x \) km away at a speed of 5 km an hour and come back at a speed of 10 km an hour.

\[ \frac{x}{5} + \frac{x}{10} \leq 1 \]

10 Express the relationships between the following quantities using an inequality.

☐ (1) A number \( x \) plus 5 is less than twice \( x \).

☐ (2) When putting \( x \) 200-g articles in a box weighing 400 g, the total weight is heavier than \( y \) g.

☐ (3) When cutting a 2-meter string to make 30-centimeter strings, what is left is longer than 40 cm.

☐ (4) It took less than 30 minutes to walk \( a \) meters, covering \( b \) meters at a speed of 50 meters per minute and the rest at a speed of 200 meters per minute.

11 Express the relationships between the following quantities using an inequality.

☐ (1) The total cost of \( a \) 80-yen stamps and \( b \) 50-yen stamps is at most 2000 yen.

☐ (2) A student's scores on three math tests were \( a \), \( b \), and \( c \) points, and the average score was at least 70.

☐ (3) When the cost price of an article is \( x \) yen and a 25% profit is expected, its tag price is at least 1800 yen.

☐ (4) The concentration of the mixture of \( x \)-gram 3% salt water and \( y \)-gram 11% salt water is at most 7%.
1. Answer the following questions.
   - (1) You bought 10 stamps, which were either 80 or 50 yen apiece. When you have $x$ 80-yen stamps, how much did you pay?
   - (2) Four students' scores on a test were $a + 5$ (points), $a$ (points), $a - 7$ (points), and $a - 2$ (points). What is their average score?
   - (3) There is a square whose sides are 10 cm. Calculate the area of a rectangle by making its length 4 cm shorter and its width $a$ cm longer.
   - (4) The length of a rectangle is $a$ cm and its width is 4 cm longer than the length. How long is the side of a square whose circumference is equal to this rectangle?
   - (5) The price of an article whose cost price is $x$ yen was set as $y$ yen, expecting a $2\ wari$ profit. Express the relationship of the quantities using an equality.
   - (6) It took at most 30 minutes to walk from home to school, covering $x$ meters at a speed of 80 meters per minute, and the rest $y$ meters at 100 meters per minute. Express the relationship of the quantities using an inequality.

2. Calculate the area of the shaded part of the following figures. Use $\pi$ for the ratio of the circumference of a circle to its diameter.
   - (1) 
   - (2) 
   - (3) 

3. As shown in the figure on the right, you can stick sheets of square paper together, partially overlapping each other. Answer the following questions.
   - (1) How many square centimeters is the total area of the figure when you stick 5 sheets of square paper together?
   - (2) How many square centimeters is the total area of the figure when you stick $n$ sheets of square paper together?

4. As shown in the calendar for a month on the right, three dates are enclosed with $\square$. Use $n$ to stand for the smallest number of the dates and write an expression to indicate the sum of the three numbers.
1 The rules of using letters Write the following expressions without using × or ÷ symbols.

☐ (1) \( a \times (-0.1) \times b \) ☐ (2) \( x \times x \times 3 \) ☐ (3) \( (a + p) \div (-5) \)

☐ (4) \( a \div 7 \times b \div x \) ☐ (5) \( a - b \times 2 \div c \) ☐ (6) \( (x + y) \times 2 \div 9 \)

2 How to express quantities Write an expression to indicate the following quantities.

☐ (1) a number divided by 6 for a quotient of \( m \) and a remainder of \( n \)

☐ (2) the payment to buy a 2000-yen article at an \( x\% \) discount

☐ (3) the distance you travel in 20 minutes at a speed of \( x \) km an hour [Unit: km]

3 The value of expressions Find the value of the following expressions.

☐ (1) \( 6(a-1) \), when \( a = -2 \)

☐ (2) \( 9x^2 - 3x \), when \( x = \frac{1}{3} \)

4 Combining terms Calculate.

☐ (1) \( 6x - x - 11x + 2x \) ☐ (2) \( 32x + 54 - 24x - 15 \)

☐ (3) \( 0.3x - 2.1 + 3.3x - 0.5 \) ☐ (4) \( \frac{3}{5} x^2 - \frac{1}{3} x + \frac{1}{2} \)

5 Calculating linear expressions Calculate.

☐ (1) \( (-2x + 3) + (4x - 6) \) ☐ (2) \( (7a - 16) - (-a - 23) \)

☐ (3) \( 2(5a - 8) + 6(3a - 5) \) ☐ (4) \( 5(x - 2) - 2(4x - 3) \)

☐ (5) \( \frac{5x - 7}{9} \times 36 \) ☐ (6) \( \left( \frac{3}{8} a - \frac{5}{6} \right) \div \left( -\frac{1}{24} \right) \)

☐ (7) \( \left( \frac{x - 2}{4} - \frac{3 + 5x}{2} \right) \times 8 \) ☐ (8) \( \frac{2x - 1}{3} - \frac{3x - 5}{4} \)
1. Calculate.

(1) \[6x + 3(3x + 8) - 5(2x - 6)\]

(2) \[0.2(5x - 6) - 9(0.6x - 2.1)\]

(3) \[6\left(\frac{1}{3}x - 2\right) + 8\left(-\frac{3}{4}x + 3\right)\]

(4) \[\frac{6x + 4}{7} \times (-35) - \frac{5x + 3}{8} \times 56\]

(5) \[\frac{3(2x + 4)}{5} + \frac{2(4x - 5)}{3}\]

(6) \[\frac{11}{12}x - \frac{3(2x - 7)}{4} + \frac{5(x + 3)}{6}\]

2. When \(x = -\frac{2}{9}\), find the value of the following expressions.

(1) \[(3x + 5) - 3(4x + 2)\]

(2) \[\frac{x + 1}{2} + \frac{2x - 1}{3}\]

3. When \(A = 3x + 5\), \(B = -2x + 7\), \(C = x - 6\), calculate the following expressions.

(1) \[2A + B - 3C\]

(2) \[\frac{A + B}{3} - \frac{C}{4}\]

4. Answer the following questions.

(1) You went to a place \(a\) km away in \(x\) hours and came back in \(y\) hours. Calculate the average speed per hour.

(2) You mixed 300 grams of \((p + 2)\%\) salt water and 200 grams of \((p - 3)\%\) salt water. How much salt is contained in this salt water?

5. As shown in the figures on the right, you can make pyramids of square cards with a side of 1 cm by arranging them one by one without gaps.

Answer the following questions.

(1) How many centimeters is the circumference of the \(n\)-th figure?

(2) How many square centimeters is the area of the \(n\)-th figure?
Chapter 3  Equations

Key points of study

1. Equations
   (1) An equality that holds true only when you substitute a particular value for a letter is called an equation. The particular value is called the solution to the equation.
   (2) Finding the solution to an equation is called solving the equation.

2. Properties of equalities
   (1) An equality holds true if the same number is added to both sides.
       \[ A = B, \text{ then } A + C = B + C \]
   (2) An equality holds true if the same number is subtracted from both sides.
       \[ A = B, \text{ then } A - C = B - C \]
   (3) An equality holds true if both sides are multiplied by the same number.
       \[ A = B, \text{ then } AC = BC \]
   (4) An equality holds true if both sides are divided by the same number.
       \[ A = B, \text{ then } \frac{A}{C} = \frac{B}{C} \quad (C \neq 0) \]
   (5) An equality holds true if both sides are exchanged.
       \[ A = B, \text{ then } B = A \]

3. Linear equations and how to solve them
   (1) You can move a term from one side of an equality to the other by changing its sign. This is called transposing the term.
   (2) If an equation can be changed into the format (a linear expression) = 0 by transposing the terms and simplifying the expressions, it is called a linear equation.
   (3) Multiplying both sides of an equation by a common multiple of the denominators to change it into an equation that does not contain fractions is called canceling the denominators.
   (4) How to solve a linear equation
       ① Remove all parentheses and change coefficients in fraction or decimal form into integers.
       ② Put the terms using the letter x on one side and the numeral terms on the other side.
       ③ Simplify both sides to make the format ax = b.
       ④ Divide both sides by a, the coefficient of x.

4. Properties of proportional expressions
   (1) \( a : b = c : d \), \( ad = bc \) (Product of the outer terms = Product of the inner terms)

5. How to solve word problems of linear equations
   (1) Decide what to find out and let x be the unknown quantity or the quantity related to it.
   (2) Find an equivalent relationship between two quantities and write an equation using expressions in terms of x and an equality sign.
   (3) Solve the equation and find the solution.
   (4) Check if the solution actually applies to the problem.
**1 Equations and how to solve them**

### Let's learn the basics

An equality that holds true only when you substitute a particular value for a letter is called an **equation**. The particular value is called the **solution to the equation**.

You can find out which of 1, 2, 3 or 4 is the solution to the equation \( x+2=3x-4 \) by checking if the equality holds true when \( x \) is substituted for the numbers.

<table>
<thead>
<tr>
<th>the value of ( x )</th>
<th>the left side</th>
<th>the right side</th>
<th>( x+2=3x-4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1+2=3</td>
<td>3x1-4=-1</td>
<td>not true</td>
</tr>
<tr>
<td>2</td>
<td>2+2=4</td>
<td>3x2-4=2</td>
<td>not true</td>
</tr>
<tr>
<td>3</td>
<td>3+2=5</td>
<td>3x3-4=5</td>
<td>true</td>
</tr>
<tr>
<td>4</td>
<td>4+2=6</td>
<td>3x4-4=8</td>
<td>not true</td>
</tr>
</tbody>
</table>

Since the equality holds true when \( x \) is substituted for 3, the solution to the equation \( x+2=3x-4 \) is 3.

### 1 About each equation below, mark a circle if 5 is the solution, or mark a cross if it isn’t.

1. \( x-4=2 \)  <br>
2. \( 3x=x+10 \)

3. \( 2x+5=3x-10 \)  <br>
4. \( 6x-2=3(x+4) \)

5. \( 7(x-3)=2x+4 \)  <br>
6. \( 4(x+2)=7(x-1) \)

### 2 About each equation below, choose the solution among 1, 2, 0, 1, 2.

1. \( 3x-1=2 \)  <br>
2. \( 5x+2=2x-4 \)

3. \( 4(x-2)+8=0 \)  <br>
4. \( 5(x+4)=3(x+6) \)

5. \( 3.1x+0.6=2.3x+1.4 \)  <br>
6. \( \frac{2x+3}{7} = \frac{3x-1}{5} \)

### 3 About each equation below, mark a circle if the value given in [ ] is the solution, or mark a cross if it isn’t.

1. \( 3x+2=8+6x \) \([-1]\)  <br>
2. \( 3(1-x)=2x-17 \) \([4]\)

3. \( 0.6x-0.4=0.3x-2.2 \) \([-6]\)  <br>
4. \( \frac{x-1}{3} = \frac{x+2}{5} \) \([5]\)
Chapter 3 Equations

Let's learn the basics 2 Properties of equalities

Finding the solution to an equation is called solving the equation. The solution to an equation in terms of \( x \) is expressed as \( x = a \).

[Question] When solving the following equation, which property of equalities shown below can be used? Fill in the blank with a number ((i)~(iv)) representing the suitable property.

\[
\begin{align*}
3x - 5 &= 7 \\
3x - 5 + 5 &= 7 + 5 \\
3x &= 12 \\
\frac{3x}{3} &= \frac{12}{3} \\
x &= 4
\end{align*}
\]

(Properties of equalities)

(i) If \( A = B \), then \( A + C = B + C \)

(ii) If \( A = B \), then \( A - C = B - C \)

(iii) If \( A = B \), then \( AC = BC \)

(iv) If \( A = B \), then \( \frac{A}{C} = \frac{B}{C} \) (\( C \neq 0 \))

[Solution] (1) If you add 5 to both sides, only letter terms remain on the left side. Here you use the property (i), that is, an equality holds true if the same number is added to both sides.

(2) If you divide both sides by 3, the coefficient of \( x \) on the left side becomes 1. Here you use the property (iv), that is, an equality holds true if both sides are divided by the same number.

[Answer] (1) (i) (2) (iv)

4 When solving the following equations, which property of equalities in Let's learn the basics 2 shown above do you use? Fill in the blank with a number((i)~(iv)) representing the suitable property.

\[
\begin{align*}
\Box (1) & \quad 2x + 7 &= 1 \\
& \quad 2x + 7 - 7 = 1 - 7 \\
& \quad 2x = -6 \\
& \quad \frac{2x}{2} = \frac{-6}{2} \\
& \quad x = -3
\end{align*}
\]

\[
\begin{align*}
\Box (2) & \quad \frac{1}{4}x - 8 &= 5 \\
& \quad \frac{1}{4}x - 8 + 8 = 5 + 8 \\
& \quad \frac{1}{4}x = 13 \\
& \quad \frac{1}{4}x \times 4 = 13 \times 4 \\
& \quad x = 52
\end{align*}
\]

5 When solving the following equations, fill in the blank with a suitable number.

\[
\begin{align*}
\Box (1) & \quad x - 3 &= 6 \\
& \quad x - 3 + [\Box (2)] &= 6 + [\Box (3)] \\
\text{that is,} & \quad x = [\Box (4)]
\end{align*}
\]

\[
\begin{align*}
\Box (2) & \quad 5x &= 20 \\
& \quad \frac{5x}{[\Box (2)]} &= \frac{20}{[\Box (3)]} \\
& \quad x &= [\Box (4)]
\end{align*}
\]

6 About the questions (1) to (4) below, explain how you transform the upper equality to the lower one.

\[
\begin{align*}
\Box (1) & \quad x + 5 &= 9 \\
& \quad x = 4
\end{align*}
\]

\[
\begin{align*}
\Box (2) & \quad x - 6 &= 2 \\
& \quad x = 8
\end{align*}
\]

\[
\begin{align*}
\Box (3) & \quad -2x &= 14 \\
& \quad x = -7
\end{align*}
\]

\[
\begin{align*}
\Box (4) & \quad \frac{x}{3} &= 2 \\
& \quad x = 6
\end{align*}
\]
Let's learn the basics on how to solve equations using the properties of equalities.

You can find the solution to an equation by transforming it using the properties of equalities.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Transformation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x - 8 = 5$</td>
<td>Add 8 to both sides.</td>
<td>$x = 13$</td>
</tr>
<tr>
<td>$x + 7 = 2$</td>
<td>Subtract 7 from both sides.</td>
<td>$x = -5$</td>
</tr>
</tbody>
</table>

Multiply both sides by 4.

$\frac{1}{4}x = 2$

$\frac{1}{4}x \times 4 = 2 \times 4$

$x = 8$

Divide both sides by 6.

$-6x = 18$

$-\frac{6x}{-6} = \frac{18}{-6}$

$x = -3$

7 Solve the following equations.

- $x + 3 = 7$
- $x + 9 = 12$
- $x + 10 = 5$
- $8 + x = 8$
- $x + 15 = 0$
- $x + 9 = 6$
- $x - 5 = 4$
- $x - 1 = 15$
- $-3 + x = 8$
- $x - 2 = -9$
- $x + 4 = -6$
- $-15 + x = 11$

8 Solve the following equations.

- $x + \frac{2}{3} = 1$
- $x - \frac{2}{7} = \frac{6}{7}$
- $x + \frac{1}{3} = -\frac{1}{2}$
- $x - 3 = 1.8$
- $3.7 + x = -1.3$
- $x + 5.3 = 2.6$

9 Solve the following equations.

- $3x = 15$
- $4x = 24$
- $6x = -18$
- $8x = 4$
- $15x = 20$
- $-9x = 27$
- $\frac{x}{6} = 3$
- $\frac{x}{4} = -3$
- $-\frac{x}{7} = 6$
- $\frac{3}{5}x = 2$
- $\frac{3}{4}x = -6$
- $-\frac{5}{8}x = \frac{10}{3}$
Let's learn the basics  How to solve equations

① You can move a term from one side of an equality to the other side by changing its sign. This is called transposing the term.

② If an equation can be changed into the format \((a\text{ linear expression})=0\) by transposing the terms and simplifying the expressions, it is called a linear equation.

③ When solving an equation, you should first change it into the form of \(ax=b\) and then divide both sides by \(a\), the coefficient of \(x\).

\[
\begin{align*}
(1) \quad 2x+6 &= -10 \\
& \quad \text{Transpose 6.} \\
& \quad 2x = -10 - 6 \\
& \quad 2x = -16 \\
& \quad x = -8
\end{align*}
\]

\[
\begin{align*}
(2) \quad 4x+3 &= x+21 \\
& \quad \text{Transpose 3 and } x. \\
& \quad 4x-x = 21 - 3 \\
& \quad 3x = 18 \\
& \quad x = 6
\end{align*}
\]

10 Solve the following equations.

- \(1) \quad 3x - 2 = 4\)  
- \(2) \quad 2x + 7 = 13\)  
- \(3) \quad 10 + 4x = 2\)

- \(4) \quad 11 - 2x = -5\)  
- \(5) \quad 4 = 16 - 3x\)  
- \(6) \quad 5 = 19 + 2x\)

11 Solve the following equations.

- \(1) \quad 5x = 12 - x\)  
- \(2) \quad 24 - 5x = 3x\)  
- \(3) \quad 3x - 30 = -2x\)

- \(4) \quad 9 - 8x = x\)  
- \(5) \quad x - 12 = 4x\)  
- \(6) \quad 5x = 6x + 7\)

12 Solve the following equations.

- \(1) \quad 2x + 7 = x + 5\)  
- \(2) \quad 5x - 4 = 2x + 11\)

- \(3) \quad 2x + 8 = 6x - 16\)  
- \(4) \quad 3x + 2 = 12 - 2x\)

- \(5) \quad -5x + 8 = 2x - 34\)  
- \(6) \quad 7x - 18 = -4x + 26\)

- \(7) \quad 6 - 10x = 30 - 4x\)  
- \(8) \quad 5x - 12 = 7x - 6\)

- \(9) \quad 2x + 15 = 8x - 15\)  
- \(10) \quad 4x - 50 = 2x + 10\)

- \(11) \quad 3 - 8x = 39 - 4x\)  
- \(12) \quad 5 - 12x = -3x + 68\)
### Let's learn the basics

#### How to solve equations containing parentheses

When solving an equation that contains parentheses, you should first remove them and then transpose the terms.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) (4(x + 3) = x + 6)</td>
<td>(4x + 12 = x + 6)</td>
</tr>
<tr>
<td>(2) (5(x - 1) = 3(2 - x) + 13)</td>
<td>(5x - 5 = 6 - 3x + 13)</td>
</tr>
</tbody>
</table>

### 13 Solve the following equations.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) (3(x + 1) - 7 = 14)</td>
<td>(9 - 4(x - 3) = 5)</td>
</tr>
<tr>
<td>(3) (5x - (2x + 3) = 6)</td>
<td>(2x + 2(3x + 8) = 0)</td>
</tr>
<tr>
<td>(5) (3(x - 5) = 1 - x)</td>
<td>(8x - 5 = 3(4x + 9))</td>
</tr>
<tr>
<td>(7) (2(3x + 1) - 3 = 8x + 5)</td>
<td>(5x - 2 = 7(2 - x) + 8)</td>
</tr>
<tr>
<td>(9) (2x - 3(x + 1) = 2 + 4x)</td>
<td>(x - 4(2x - 7) = 3x - 12)</td>
</tr>
</tbody>
</table>

### 14 Solve the following equations.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) (8(x - 1) + 2(x - 6) = 0)</td>
<td>(5(x + 3) - 8(3x - 10) = 0)</td>
</tr>
<tr>
<td>(3) (4(x - 2) + 3(5 - x) = 6)</td>
<td>(6(x - 8) - (4 - x) = -3)</td>
</tr>
<tr>
<td>(5) (2(4x - 1) = 3(x + 6))</td>
<td>(4(x - 3) = 9 - 5(1 - x))</td>
</tr>
<tr>
<td>(7) (7(x - 1) = 4(2x - 5) + 13)</td>
<td>(3 - 8(x - 6) = 5(3 - x))</td>
</tr>
<tr>
<td>(9) (3x - 7(x - 1) = -3(2x + 3))</td>
<td>(8x - 2(3x + 5) = 3x - 5(x - 10))</td>
</tr>
</tbody>
</table>

### 15 Solve the following equations.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) (4(x - 2) - 3(x - 3) = 2(x - 4) + 6)</td>
<td>(10x - 8(3 + x) = 7(2x - 5) - 5(2x - 3))</td>
</tr>
</tbody>
</table>
Let's learn the basics

How to solve equations containing decimals

When solving an equation that contains decimals, you should first change them into integers by multiplying both sides by powers of 10 (10, 100, \ldots).

1. \(0.7x + 0.4 = 3.2\)
   - Multiply both sides by 10.
   - \(7x + 4 = 32\)
   - \(7x = 32 - 4\)
   - \(7x = 28\)
   - \(x = 4\)

2. \(0.3x - 0.08 = 0.02x + 0.2\)
   - Multiply both sides by 100.
   - \(30x - 8 = 2x + 20\)
   - \(28x = 28\)
   - \(x = 1\)

16 Solve the following equations.

- (1) \(0.2x - 0.5 = 1.3\)
- (2) \(0.4x + 3 = 4.6\)
- (3) \(7 - 0.3x = 2.5\)
- (4) \(0.2x - 0.3 = 0.5x + 0.9\)
- (5) \(0.1x + 0.4 = 0.3x - 0.6\)
- (6) \(3.5x - 0.9 = 2.8x + 1.2\)
- (7) \(1.2x + 5 = 1.8x - 0.4x\)
- (8) \(1.6 - 1.1x = 4 - 0.5x\)

17 Solve the following equations.

- (1) \(0.02x + 0.09 = 0.05\)
- (2) \(0.15x - 0.49 = 0.11\)
- (3) \(0.36 - 0.07x = 0.05x\)
- (4) \(0.32x + 0.14 = 0.07x + 0.64\)
- (5) \(0.13x + 0.4 = 2.2 - 0.17x\)
- (6) \(0.04x + 0.3 = -0.16x - 0.7\)
- (7) \(0.2x + 1 = 0.28 - 0.16x\)
- (8) \(0.14x - 3 = 0.1x - 1.8\)

18 Solve the following equations.

- (1) \(0.3(5x + 8) = 1.3x\)
- (2) \(0.4(x + 3) = x - 3\)
- (3) \(0.05(3x - 8) = 0.18x - 0.13\)
- (4) \(0.08(x - 4) - 0.1 = 0.5x\)
- (5) \(1.3x - 0.8(x - 1.5) = 2.7\)
- (6) \(0.6(x - 0.7) = 0.7(0.9x + 0.3)\)
1. Let's learn the basics How to solve equations containing fractions

When solving an equation that contains fractions, you should first change them into integers by multiplying both sides by the least common multiple of the denominators. This transformation is called canceling the denominators.

\[
\frac{2}{3}x - \frac{1}{2} = \frac{3}{4}x
\]

Multiply both sides by 12, the least common multiple of the denominators.

\[
\frac{2}{3}x \times 12 - \frac{1}{2} \times 12 = \frac{3}{4}x \times 12
\]

Cancel the denominators to change the coefficients into integers.

\[
x = 6
\]

19 Solve the following equations.

(1) \( \frac{1}{4}x + 2 = 7 \)

(2) \( \frac{2}{5}x - 3 = \frac{1}{2} \)

(3) \( 3 = \frac{5}{6}x - \frac{9}{2} \)

(4) \( \frac{1}{3}x = \frac{1}{4}x + \frac{5}{6} \)

(5) \( \frac{1}{3}x - 1 = \frac{2}{5}x + 2 \)

(6) \( \frac{5}{6}x + \frac{3}{8} = \frac{2}{3}x - \frac{3}{2} \)

(7) \( \frac{3}{4}x + \frac{2}{3} - \frac{3}{2} = \frac{1}{3}x \)

(8) \( \frac{1}{6}x - 1 = \frac{3}{8}x - \frac{4}{9} \)

(9) \( \frac{1}{2}(5x - 3) = 1 \)

(10) \( \frac{1}{3}(2x - 1) = \frac{2}{7}(3x - 2) \)

20 Solve the following equations.

(1) \( \frac{x}{3} = \frac{x - 2}{5} \)

(2) \( \frac{2x + 1}{5} = \frac{x - 7}{4} \)

(3) \( \frac{x + 3}{2} = \frac{x - 2}{3} - 1 \)

(4) \( \frac{2x + 5}{3} = \frac{4x + 2}{5} = 1 \)

(5) \( \frac{3(x - 4)}{2} + 2(3x - 4) = \frac{1}{2} \)

(6) \( \frac{x - 3}{2} + \frac{3x - 1}{4} = \frac{5x - 7}{3} \)

21 Solve the following equations.

(1) \( \frac{x + 1}{5} + 0.25x = -\frac{1}{4} \)

(2) \( \frac{1}{3}(x - 2) + 0.5(x + 2) = 12 \)
### Let's learn the basics

How to solve equations using proportional expressions

An expression in which an equal sign joins two equal ratios is called a **proportional expression**.

**Property of proportional expressions:**

If \( a : b = c : d \), then \( ad = bc \). (Product of the outer terms = Product of the inner terms)

You can use the property shown above to solve proportional expressions.

<table>
<thead>
<tr>
<th>Inner terms</th>
<th>Inner terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) ( x : 3 = 6 : 9 )</td>
<td>(2) ( (x+2) : 4 = x : 3 )</td>
</tr>
<tr>
<td>Outer terms</td>
<td>Outer terms</td>
</tr>
<tr>
<td>( x \times 9 = 3 \times 6 )</td>
<td>( (x+2) \times 3 = 4 \times x )</td>
</tr>
<tr>
<td>9x = 18</td>
<td>3x + 6 = 4x</td>
</tr>
<tr>
<td>x = 2</td>
<td>(-x = -6)</td>
</tr>
<tr>
<td></td>
<td>( x = 6 )</td>
</tr>
</tbody>
</table>

### 22 Solve the following proportional expressions.

| □ (1) \( x : 2 = 4 : 1 \) | □ (2) \( x : 3 = 3 : 1 \) | □ (3) \( x : 6 = 4 : 8 \) |
| □ (4) \( 5 : x = 10 : 4 \) | □ (5) \( 2 : x = 6 : 9 \) | □ (6) \( 3 : x = 7 : 21 \) |
| □ (7) \( 3 : 4 = x : 8 \) | □ (8) \( 3 : 15 = x : 5 \) | □ (9) \( 6 : 3 = 4 : x \) |

### 23 Solve the following proportional expressions.

| □ (1) \( (x+5) : 2 = 5 : 1 \) | □ (2) \( (x+6) : 4 = 2 : 1 \) | □ (3) \( 3 : (x-2) = 1 : 3 \) |
| □ (4) \( (x-6) : 1 = x : 2 \) | □ (5) \( (x+4) : 15 = x : 5 \) | □ (6) \( 4 : x = 8 : (x+5) \) |
| □ (7) \( (x+1) : 4 = (x+2) : 6 \) | □ (8) \( 15 : (x+3) = 9 : (2x-1) \) | □ (9) \( (x-1) : (2x-3) = 12 : 18 \) |

### 24 Solve the following proportional expressions.

| □ (1) \( x : 2 = 0.3 : 0.6 \) | □ (2) \( 3 : x = 1.5 : 2.5 \) |
| □ (3) \( x : 2.8 = (x+1) : 4.2 \) | □ (4) \( (x-2) : 0.2 = (x+3) : 0.4 \) |
| □ (5) \( 1.8 : (x+1) = 0.6 : (2x-3) \) | □ (6) \( (x+4) : (3x-1) = 1.8 : 1.5 \) |
| 1 | Solve the following equations. |
|---|---|---|
| (1) | $x+8=11$ | (2) | $x-4=9$ | (3) | $x+0.6=0.1$ |
| (4) | $6x=24$ | (5) | $9x=-54$ | (6) | $\frac{2}{7}x=-6$ |

| 2 | Solve the following equations. |
|---|---|---|
| (1) | $5x-8=2$ | (2) | $7x=3x-12$ | (3) | $3x+4=x-6$ |
| (4) | $8x+9=2x-33$ | (5) | $4x-8=7x+16$ | (6) | $-2x+8=3x-37$ |
| (7) | $3(x+2)=2x+3$ | (8) | $5x-(7x-5)=-3$ | (9) | $6x-5(2x-4)=3x-1$ |

| 3 | Solve the following equations. |
|---|---|---|
| (1) | $0.3x-0.4=2.3$ | (2) | $0.8x-0.9=x+1.5$ |
| (3) | $0.12x+0.1=0.06x-0.08$ | (4) | $0.2x-0.3(0.1x+0.2)=0.28$ |
| (5) | $\frac{4}{7}x+\frac{1}{2}=-\frac{1}{4}$ | (6) | $\frac{1}{4}x+8=\frac{2}{3}x+3$ |
| (7) | $\frac{x-1}{3}=\frac{x+1}{5}$ | (8) | $\frac{2x+1}{5}=\frac{x-7}{10}=-6$ |
| (9) | $\frac{x-5}{3}+\frac{3}{2}=\frac{3x-1}{4}$ | (10) | $\frac{x-2}{3}+0.5x=\frac{1}{6}$ |

| 4 | Solve the following proportional expressions. |
|---|---|---|
| (1) | $10:x=5:4$ | (2) | $x:6=(x+2):10$ |
| (3) | $3:(x-3)=15:(x+1)$ | (4) | $(x-2):(2x+1)=0.3:2.1$ |
Let's learn the basics  Problems about the values for letters

When an equation in terms of $x$ contains another letter, transform it into an equation in terms of the letter by substituting the given solution for $x$ and solve it to find the value for the letter.

**Question**
When the solution to the equation in terms of $x$, $3x + a = x - a$, is $x = 2$, find the value for $a$.

**Solution**
Since the solution to the equation is $x = 2$, the equality holds true when you substitute 2 for $x$.

$3 \times 2 + a = 2 - a$

This is an equation in terms of $a$, so it can be solved this way: $6 + a = -2$, $2a = -4$, $a = -2$

**Answer**
$a = -2$

1. Answer the following questions.

   1. When the solution to the equation in terms of $x$, $5x + a = x + 8$, is $x = 3$, find the value for $a$.

   2. When the solution to the equation in terms of $x$, $4(x + a) = x + 6$, is $x = -2$, find the value for $a$.

   3. When the solution to the equation in terms of $x$, $2x - a = 0.5x + 2$, is $x = 4$, find the value for $a$.

   4. When the solution to the equation in terms of $x$, $\frac{x - a}{3} = \frac{x + 2a}{4}$, is $x = 10$, find the value for $a$.

2. Fill in the blanks.

   1. The solution to the equation $12 + \square x = 6x$ is $x = 4$.

   2. The solution to the equation $\frac{x + \square}{2} = \frac{1}{3} x + 3$ is $x = 12$.

3. Answer the following questions about the equation in terms of $x$, $a(x - 3) - 3ax = a + 16$.

   1. When $a = 2$, solve this equation.

   2. When the solution to this equation is $x = -4$, find the value for $a$.

4. When the solution to the equation in terms of $x$, $5(x + 3) - 2(x + 2a) = 48$, is equal to the solution to the equation $0.8x - 0.3 = 0.5x - 4.2$, find the value for $a$. 
Let's learn the basics 2 Problems about numbers

Question 1 Three times a number $x$ minus 8 is 13. Find the number $x$.

Solution You can write an equation in terms of $x$, $3x - 8 = 13$.
Solving this, you get $3x = 21$, so $x = 7$.

Answer $x = 7$

Question 2 The sum of three consecutive integers is 48. Find these integers.

Solution Letting the second greatest number be $x$, the three integers can be expressed as $x - 1$, $x$, and $x + 1$. So, $(x-1) + x + (x+1) = 48$.
Solving this, you get $3x = 48$, so $x = 16$. Therefore the three integers are 15, 16, and 17.

Answer 15, 16 and 17

5 Answer the following questions.

□(1) Four times a number $x$ minus 7 is 25. Find the number $x$.

□(2) Four times a number plus 11 is 4 less than seven times the original number. Find the original number.

□(3) Three times the sum of a number and 2 is 18 more than five times the original number. Find the original number.

□(4) Five times $x$ divided by 32 is 3 with a remainder of $x$. Find the value for $x$.

6 Answer the following questions.

□(1) The sum of three consecutive integers is 108. Find these integers.

□(2) The sum of four consecutive integers is 254. Find these integers.

□(3) The sum of three consecutive even numbers is 162. Find these numbers.

7 Answer the following questions.

□(1) There is a natural number whose ones digit is 8. When you exchange its ones digit and tens digit, the new number is 27 more than the original number. Find the original number.

□(2) There is a two-digit number. The sum of its ones digit and tens digit is 13. When you exchange its ones digit and tens digit, the new number is 45 more than the original number. Find the original number.
Let's learn the basics  3  Problems about prices and the number of items

[Question] You paid 1210 yen for 12 items, which were some 80-yen pencils and some 130-yen ball-point pens. How many pencils and how many ball-point pens did you buy?

[Solution] When you let \( x \) be the number of pencils you bought, the number of ball-point pens you bought is \( 12 - x \).

\[
80x + 130(12 - x) = 1210
\]

Solving this, you get \( x = 7 \). Therefore, the number of pencils is 7 and the number of ball-point pens is \( 12 - 7 = 5 \).

[Answer] Pencils...7, Ball-point pens...5

8 Answer the following questions.

1. You paid 500 yen for 6 pencils and received 50 yen in change. How much was 1 pencil?

2. The price of 8 Japanese cakes with a 80-yen box is 1200 yen. How much is 1 cake?

3. A 40-yen box filled with some 60-yen tangerines costs just 1000 yen. How many tangerines are there in the box?

9 Answer the following questions.

1. You paid 680 yen for 10 items, which were some 50-yen postcards and some 80-yen stamps. How many postcards and how many stamps did you buy?

2. You paid 940 yen for 12 desserts, which were some 70-yen puddings and some 90-yen coffee jellies. How many puddings and how many coffee jellies did you buy?

3. You paid 1320 yen for a 100-yen box filled with 16 pieces of fruit, which were some 120-yen apples and some 50-yen tangerines. How many apples and how many tangerines did you buy?

4. You paid 500 yen for 50 sheets of some 6-yen colored paper and some 15-yen cardboard and received 20 yen in change. How many sheets of colored paper and how many sheets of cardboard did you buy?

Point

1. How to solve word problems of equations
   (1) Make sure what to find out and let \( x \) be the unknown quantity or the quantity relevant to it.
   (2) Find an equivalent relationship between two quantities and write an equation using expressions in terms of \( x \) and an equality sign.
   (3) Solve the equation to find the solution, and check if it actually applies to the problem.
Let's learn the basics 4 Problems about overs and shorts

Question You are going to distribute some tangerines equally to some children. If each child gets 5 tangerines, there are 4 short. If they get 4 tangerines, there are 2 left over. How many children and how many tangerines are there?

Solution Letting \( x \) be the number of children, you can write two expressions to indicate the number of tangerines.

"If each child gets 5 tangerines, there are 4 short," so there are \( 5x - 4 \) (tangerines).

"If they get 4 tangerines, there are 2 left over," so there are \( 4x + 2 \) (tangerines).

Since these two expressions stand for the same number, \( 5x - 4 = 4x + 2 \).

Solving this equation, you get the solution \( x = 6 \). Therefore, there are 6 children and 26 (=\( 5 \times 6 - 4 \)) tangerines.

Answer Children...6, Tangerines...26

Solve this problem by writing two expressions to indicate the number of children, letting \( x \) be the number of tangerines.

10 Answer the following questions.

\( \square \) (1) You are going to distribute some apples equally to some children. If each child gets 3 apples, there are 5 left over. If each of them gets 4 apples, there are 10 short. How many children are there?

\( \square \) (2) You are going to distribute some candies equally to some children. If each child gets 12 candies, there are 20 left over. If each of them gets 14 candies, there are 4 left over. How many children are there?

\( \square \) (3) You were going to buy a certain number of 130-yen notebooks, but you were 300 yen short. So you tried to buy the same number of 115-yen notebooks, but you were still 135 yen short. How many notebooks were you going to buy?

11 Answer the following questions.

\( \square \) (1) You are going to distribute some pencils equally to the students of your class. If each of them gets 5 pencils, there are 20 short. If each of them gets 4 pencils, there are 16 left over. How many students and how many pencils are there?

\( \square \) (2) You are going to collect money from the attendants of a class meeting. If you collect 250 yen per head, there is 950 yen left over. If you collect 230 yen per head, there is 250 yen left over. How many students are going to attend, and how much is the expense of the class meeting?

\( \square \) (3) There are some benches in the auditorium. When 8 students sat on each bench, 5 students had no seat. When 9 students sat on each bench, there were 8 vacant seats left. How many benches and how many students are there?

\( \square \) (4) There are some microscopes. At first every 3 students were told to use 1 microscope, but 2 students couldn’t use any. Then every 4 students were told to use 1 microscope. As a result, the last microscope was used by only 2 students and 2 microscopes were left unused. How many microscopes and how many students are there?
Let's learn the basics  

**Question**
A boy has 24 pencils and his brother has 8 pencils. The boy wants to give some pencils to his brother so that he has 10 more than his brother. How many pencils should the boy give to his brother?

**Solution**
If the boy gives $x$ pencils to his brother, he gets $24-x$ pencils and his brother gets $8+x$ pencils.

\[
\text{(The boy's pencils)} = \text{(His brother's pencils)} + 10,
\]

so you can write the following equation:

\[
24 - x = (8 + x) + 10
\]

Solving this, you get the solution $x = 3$. Therefore, the boy should give 3 pencils to his brother.

**Answer**
3 pencils

---

12 Answer the following questions.

1) A boy has 42 postcards and his brother has 10 postcards. The boy wants to give some postcards to his brother so that he has 20 more than his brother. How many postcards should he give to his brother?

2) A girl made 41 cookies and her sister made 13 cookies. The girl wants to give some cookies to her sister so that she has twice as many as her sister. How many cookies should she give to her sister?

3) A and B are going to cut a 2-meter ribbon to share two pieces of ribbon. When they cut the ribbon so that A’s piece is 12 cm longer than B’s, how many cm is A’s piece and how many cm is B’s piece?

---

Let's learn the basics  

**Question**
There are some apples and tangerines. The ratio of the number of apples to the number of tangerines is 4 to 5, and the number of apples is 16. Find the number of tangerines.

**Solution**
If you let $x$ be the number of tangerines, $4:5 = 16:x$.

This proportional expression can be solved as follows: $4x = 5 \times 16$, $4x = 80$, $x = 20$

**Answer**
20

---

13 Answer the following questions.

1) There are some 50-yen and 80-yen stamps. The ratio of the number of 50-yen stamps to the number of 80-yen stamps is 3 to 5, and the number of 50-yen stamps is 18. Find the number of 80-yen stamps.

2) The ratio of the number of boys to girls at a junior high school is 13 to 12, and the number of girls is 252. Find the number of boys.

3) There are two different positive numbers. The difference between them is 6, and the ratio of the smaller number to the larger number is 2 to 3. Find the smaller number.
Let's learn the basics 7 Problems about age and average

Question 1  A man is 40 and his son is 8 now. In how many years will the man be 3 times as old as his son?

Solution  Suppose the man will be 3 times as old as his son in \( x \) years.
In \( x \) years he will be \( 40 + x \) years old and his son will be \( 8 + x \) years old, so \( 40 + x = 3(8 + x) \).
Solving this, you get \( x = 8 \). Therefore, the man will be 3 times as old as his son in 8 years.

Answer  In 8 years

Question 2  A’s score and B’s score on a math test is 84 points and 76 points, respectively. If C’s score is added, the three students’ average score becomes 86 points. What is C’s score?

Solution  If you let \( x \) be C’s score, since (Three students’ total score) = (Their average score) \( \times \) 3,
so \( 84 + 76 + x = 86 \times 3 \).
Solving this, you get \( x = 98 \). Therefore, C’s score is 98 points.

Answer  98 points

14 Answer the following questions.

☐1) A woman is 35 and her child is 5 now. In how many years will the woman be 4 times as old as her child?

☐2) A 46-year-old man has three children aged 16, 18, and 20. How many years ago was the sum of the three children’s ages equal to the man’s age?

☐3) A boy has saved 3500 yen and his brother has saved 1700 yen. Next month they are going to start saving a certain amount of money every month. If the boy saves 100 yen a month and his brother saves 250 yen a month, in how many months will their savings become equal?

☐4) A has saved 9500 yen and B has saved 5500 yen. Next month they are going to start withdrawing 500 yen every month and spend it. In how many months will A’s savings become three times B’s?

15 Answer the following questions.

☐1) Student A got 70, 66, and 72 points on the last three math tests. How many points does A have to get on the next test to make his average score of the four tests 75 points?

☐2) The average height of A, B, C, and D is 152.5 cm. If E’s height is added, the five students’ average height becomes 152 cm. What is E’s height?

☐3) All of the 25 students in a class had their weight measured. The boys’ average was 47 kg, the girls’ average was 42 kg, and the class average was 44.8 kg. How many boys are there in this class?
1. Answer the following questions.
   □ (1) When the solution to the equation in terms of $x$, $2ax - 7 = 4a - 7x$, is $-5$, find the value for $a$.
   □ (2) The sum of three consecutive odd numbers is 63. Find these numbers.
   □ (3) As shown in the calendar of a month on the right, you can enclose nine dates arranged 3 by 3 in [ ]. In this example, the sum of the nine numbers enclosed in [ ] is 90, and the date in the center is 10. When the sum of the nine numbers is 207, what is the number in the center?

2. Answer the following questions.
   □ (1) You bought 11 shortcakes and paid 2590 yen, including the price of the box. When the box is 60 yen, how much is 1 shortcake?
   □ (2) You paid 1500 yen for 18 items, some 60-yen pencils and some 90-yen ball-point pens, and received 240 yen in change. How many pencils did you buy?
   □ (3) A school has 197 first-year students, consisting of boys and 5 fewer girls than boys. How many first-year boys are there in this school?
   □ (4) A boy and his brother have 5600 yen in total, and the boy has three times as much as his brother. How much does the boy have and how much does his brother have?

3. Answer the following questions.
   □ (1) You are going to distribute some sheets of drawing paper equally to some children. If each child gets 8 sheets, the last one is given only 5 sheets. If they get 7 sheets, there are 4 sheets left over. How many children and how many sheets of drawing paper are there?
   □ (2) There is a man aged 39. His wife is aged 33. They have three children aged 12, 10, and 6. When the sum of the couple’s ages becomes twice the sum of the children’s ages, how old will the man be?
   □ (3) On a three-day weekend in a month, a botanical garden had 1460 visitors a day on average. The visitors on the second day were 240 fewer than those on the first day and 150 more than those on the third day. How many people visited the botanical garden on the first day?
3 Using equations (2)

Let's learn the basics 1 Problems about speed (1)

**Question:** A girl left home for the station. 6 minutes later, her brother realized she forgot something and rode his bicycle to catch up with her. The girl walked at a speed of 60 m per minute, and the boy rode at 150 m per minute. How many minutes after the boy left the house did he catch up with her?

**Solution:** Let \( x \) be the number of minutes it took the boy to catch up with his sister. The amount of time his sister walked can be expressed as \( 6 + x \) (minutes), and

\[
(\text{The boy's travel distance}) = (\text{His sister's travel distance})
\]

so,

\[
150x = 60(6 + x)
\]

Solving this, you get \( x = 4 \).

Therefore, the boy caught up with his sister 4 minutes after he left the house.

**Answer:** 4 minutes later

---

1 Answer the following questions.

☐ (1) A boy left home and headed for the station at a speed of 40 m per minute. His brother started 9 minutes later to catch up with him and traveled at 100 m per minute. How many minutes after the brother left the house did he catch up with the boy?

☐ (2) A child left home for the station at 7 o'clock. Realizing he forgot something, his father left home at 7:18 and drove to catch up with him. The child walked at a speed of 60 m per minute, and the father drove at 600 m per minute. How many minutes after the father left the house did he catch up with the child?

☐ (3) A girl left home for a park at 8 o'clock and walked at a speed of 50 m per minute. Her sister started 16 minutes later and rode her bicycle to catch up with her. She rode at 250 m per minute. What time did she catch up with the girl?

---

2 Answer the following questions.

☐ (1) At 9 o'clock a boy left home for a gym 2 km away. He walked at a speed of 80 m per minute. His brother started 16 minutes later and rode his scooter to catch up with the boy. He rode at 400 m per minute. What time and how many meters away from the gym did he catch up with the boy?

☐ (2) At 7:30 a girl left home to walk to her school 1.2 km away. 3 minutes later her brother started to walk to the school and caught up with her on the way. The girl walked at a speed of 60 m per minute, and her brother walked at 75 m per minute. What time and how many meters away from the school did he catch up with her?
Chapter 3 Equations

Let’s learn the basics 2 Problems about speed (2)

- Two people going around a pond in the opposite directions
  When they meet, the sum of their travel distance is equal to the circumference of the pond.

- Two people going around a pond in the same direction
  When one person catches up with the other, the difference between their travel distance is equal to the circumference of the pond.

3 Answer the following questions.

☐ (1) There is a pond whose circumference is 2700 m. A boy and his brother simultaneously started at the same point and walked around it in opposite directions. The boy walked at a speed of 90 m per minute, and his brother walked at 60 m per minute. How many minutes later did they first meet?

☐ (2) There is a pond whose circumference is 4.2 km. A and B simultaneously started at the same point and traveled around it in opposite directions. A ran at a speed of 200 m per minute, and B walked at 80 m per minute. How many minutes later did they first meet?

4 Answer the following questions.

☐ (1) There is a running track whose circumference is 400 m. A and B simultaneously started at the same point and ran around it in the same direction. A ran at a speed of 250 m per minute, and B ran at 200 m per minute. How many minutes later did A first catch up with B?

☐ (2) There is a lake whose circumference is 4900 m. A boy and his brother simultaneously started at point A and walked around it in the same direction. The boy walked at a speed of 95 m per minute, and his brother walked at 60 m per minute. How many hours and how many minutes later did the boy first catch up with his brother?

5 There are two towns, A and B, located 170 km apart. Mr. Matsumura left Town A for Town B and drove his car at a speed of 50 km per hour. 15 minutes later Mr. Ichikawa left Town B for Town A and rode his motorcycle at 40 km per hour. How many hours after Mr. Matsumura started did they first meet?

<table>
<thead>
<tr>
<th>Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>① The relationships between speed, time, and travel distance: (Speed) = \frac{(Travel distance)}{(Time)}, (Time) = \frac{(Travel distance)}{(Speed)}, (Travel distance) = (Speed) \times (Time).</td>
</tr>
</tbody>
</table>

82
Let's learn the basics  Problems about speed

Suppose you walk to a station 2 km away from your house. If you walk at a speed of 60 m per minute to Point A on the way and then walk to the station at 80 m per minute, you will get there in 30 minutes. How many meters away is Point A from your house?

Solution
Let \( x \) be the distance between your house and Point A. As shown in the figure on the right, the sum of the amounts of time it takes you to walk from your house to Point A and to walk from there to the station is 30 minutes.

So,

\[
\frac{x}{60} + \frac{2000-x}{80} = 30
\]

Solving this equation, you get \( x = 1200 \). Therefore, the distance between your house and Point A is 1200 m.

Answer 1200 m

Solve the problem above by letting \( x \) be the amount of time it takes you to walk from your house to Point A.

6 Answer the following questions.

1) Suppose you walked to a library 1200 m away from your house. You walked to a mailbox on the way at a speed of 60 m per minute, and then walked to the library at 50 m per minute. You got to the library 21 minutes after you left your house. How many meters away is the mailbox from your house?

2) Suppose you walked to your school 1.5 km away from your house. You walked to a station on the way at a speed of 70 m per minute, and then walked to the school at 80 m per minute. You got to the school 20 minutes after you left your house. How many meters away is the station from your house?

3) Suppose you walked from your house to a bus stop and went to the train station by bus. The total travel distance was 30 km, and you got to the station 1 hour and 27 minutes after you left your house. You walked at a speed of 4 km per hour, the bus ran at 40 km per hour, and you waited for the bus at the bus stop for 15 minutes. How many kilometers away is the bus stop from your house?

7 Answer the following questions.

1) Suppose you go from Point A to Point B either by riding a bicycle at a speed of 15 km per hour or by driving a car at 60 km per hour. It takes 1 hour more to go by bicycle than by car. How many kilometers apart are the two points?

2) Suppose you go from Point A to Point B by bicycle. If you ride at a speed of 20 km per hour, it takes 5 minutes less than riding at 18 km per hour. How many kilometers apart are the two points?

3) A boy and his brother went from their house to Point A by bicycle. The boy left the house and rode at a speed of 12 km per hour. His brother started 10 minutes later and rode on the same road at 18 km per hour. They arrived at the same time. How many kilometers away is Point A from their house?

8 It took a train 21 seconds to cross a 240-m railroad bridge, and it took 63 seconds to go through a 1080-m tunnel. The train starts to cross the bridge when its front edge gets on one edge of the bridge and finishes crossing when its tail edge gets off the other edge of the bridge. The same is true of going through a tunnel. How long is the train?
Problems about profit

1. The relationships between the cost price, the tag price and the discount price:

\[(\text{Tag price}) = (\text{Cost price}) \times (1 + (\text{Profit rate}))\] \[(\text{Selling price}) = (\text{Tag price}) \times (1 - (\text{Discount rate}))\]

2. Percentages and 百分 is expressed as decimals or fractions. 1% is 0.01 or \(\frac{1}{100}\). 1 百分 is 0.1 or \(\frac{1}{10}\).

Question: There is an article whose tag price is 1300 yen. If you sell it at a 10% discount, the profit is 17% of its cost price. How many yen is the cost price?

Solution: Let \(x\) be the cost price of this article.

\[(\text{Selling price}) - (\text{Cost price}) = (\text{Profit})\]

The selling price is the tag price (1300 yen) with a 10% discount, so

\[1300 \times (1 - 0.1) = 1170 \text{ yen} \]

Thus, \[1170 - x = 0.17x\]

Solving this equation, you get \(x = 1000\). Therefore, the cost price of this article is 1000 yen.

Answer: 1000 yen

9 Answer the following questions.

\[1\] There is an article whose tag price is 2250 yen, which is 25% higher than its cost price. What is the cost price of this article?

\[2\] You paid 2080 yen for an article at a 20% discount. What was its tag price?

\[3\] You paid 600 yen for an article whose selling price was its tag price with a 3 百分 discount minus 30 yen. What was the tag price of this article?

\[4\] You bought 10 apples at a 1 百分 5 百分 discount. You paid with 1500 yen and received 310 yen in change. How many yen was the tag price of each apple?

10 Answer the following questions.

\[1\] If you sell an article at the tag price, the profit is 25% of its cost price. You sold it at 280 yen lower than its tag price, you got a profit of 420 yen. What was the cost price of this article?

\[2\] Suppose you fix the tag price of an article whose cost price is 4000 yen. You are going to sell it at a 15% discount so that you get a profit of 19% of its cost price. How many yen should the tag price be?

\[3\] You fixed the tag price of an article, expecting a profit of 3 百分 of its cost price. If you sell it at a 2 百分 discount, you get a profit of 880 yen. What was the cost price of this article?

\[4\] You fixed the tag price of an article, expecting a profit of 25% of its cost price. At a sale, however, you sold it at 360 yen lower than the tag price and got a loss of 5% of its cost price. How many yen was the cost price?
Let's learn the basics | Problems about salt water

When solving problems about salt water, express concentration (%) as fractions or decimals.

\[
\text{Amount of salt} = \text{(Amount of salt water)} \times \left( \frac{\text{Concentration of salt water}}{100} \right)
\]

\[
\text{Concentration of salt water} = \frac{\text{(Amount of salt)}}{\text{(Amount of salt water)}} \times 100(\%)
\]

Question: Suppose you want to make salt water whose concentration is 8%, if you mix 360 g of 10% salt water and some 5% salt water, how many grams of 5% salt water do you need?

Solution: Let \( x \) be the amount of 5% salt water you need. Make a table as shown on the right and write an equation, focusing on the relationship between the amounts of salt.

\[
360 \times \frac{10}{100} + \frac{5}{100} x = \frac{8}{100} (360 + x)
\]

Solving this, you get \( x = 240 \). Therefore, you need 240 g of 5% salt water.

Answer: 240 g

11 Answer the following questions.

1. In order to make salt water whose concentration is 7%, how many grams of 3% salt water do you have to add to 300 g of 8% salt water?

2. In order to make salt water whose concentration is 8%, how many grams of 10% salt water do you have to add to 600 g of 4% salt water?

3. In order to make salt water whose concentration is 10%, how many grams of 7% salt water do you have to add to 120 g of 12% salt water?

12 Answer the following questions.

1. You mixed 10% salt water and 5% salt water to make 300 g of 7% salt water. How many grams of 10% salt water did you use?

2. You mixed 15% salt water and 8% salt water to make 630 g of 10% salt water. How many grams of 15% salt water and how many grams of 8% salt water did you use?

3. You mixed 7% salt water and 15% salt water to make 480 g of 12% salt water. How many grams of 7% salt water and how many grams of 15% salt water did you use?

13 Answer the following questions.

1. In order to make 4% salt water by adding water to 600 g of 10% salt water, how many grams of water do you need?

2. In order to make 10% salt water by adding water to 500 g of 12% salt water, how many grams of water do you need?

3. To change the concentration of 500 g of salt water from 3% to 5%, how many grams of water do you have to evaporate?

4. To change the concentration of 360 g of salt water from 3% to 10%, how many grams of salt do you have to add?
Let's learn the basics 6 Problems about changes in number

**Question** Last year there were 1200 boy and girl students in a junior high school. This year the number of boys has increased by 10% and the number of girls has decreased by 15%, and there are 25 fewer students than last year. How many boys and how many girls are there in this school this year?

**Solution** If you let \( x \) be last year's number of boys, last year's number of girls is expressed as \( 1200 - x \).

This year's number of boys is expressed as \( x \times (1 + 0.1) = 1.1x \).

This year's number of girls is expressed as \( (1200 - x) \times (1 - 0.15) = 0.85(1200 - x) \).

So, \( 1.1x + 0.85(1200 - x) = 1200 - 25 \). Solving this equation, you get \( x = 620 \).

This year's number of boys is 620 \( \times 1.1 = 682 \), This year's number of girls is \( (1200 - 620) \times 0.85 = 493 \).

(Another solution) Focusing on the difference between last year's and this year's number of boys, you can write the following equation: \( 0.1x - 0.15(1200 - x) = -25 \). Solving this, you get the same answer.

**Answer** Boys...682, Girls...493

---

14 Answer the following questions.

□1) A factory makes two products A and B. Last week it produced 3000 pieces in total. This week it produced 3170 pieces by increasing the output of A by 15% and decreasing the output of B by 5%. How many pieces of A and how many pieces of B did the factory produce last week?

□2) Last year there were 1400 boy and girl students in a junior high school. This year the number of boys has increased by 20% and the number of girls has decreased by 10%, and there are 40 more students than last year. How many boys and how many girls are there in this school this year?

□3) Last year there were 5000 male and female employees in a company. This year the number of men has decreased by 20% and the number of women has increased by 30%, and there are 10% fewer employees than last year. How many men and how many women are there in this company this year?

---

Let's learn the basics 7 Problems about figures

**Question** There is a rectangle of 5 cm long and 8 cm wide. You made the length 4 cm greater and the width a certain number of cm greater, and the area increased by 50 cm². How many centimeters greater did the width become?

**Solution** Let \( x \) cm be the increase in width. The length of the new rectangle is 5+4=9 (cm), and its width is 8+x (cm). \( \text{Area of a rectangle} = \text{Length} \times \text{Width} \), so you can write the following equation:

\[
9(8+x) = 5 \times 8 + 50
\]

Solving this, you get \( x = 2 \). Therefore, the width became 2 cm greater.

**Answer** 2 cm

15 Answer the following questions.

□1) There is a triangle with a base of 9 cm. Its area is 36 cm². How many centimeters is its height?

□2) The width of a rectangle is three times its length, and its circumference is 48 cm. How long and how wide is this rectangle?

□3) There is a rectangle whose circumference is 90 cm. Its length is \( \frac{1}{5} \) of its width plus 3 cm. Find the area of this rectangle.
Let's learn the basics

Problem about a moving point in a figure

Question: In triangle ABC on the right, angle B is a right angle, AB = 7 cm, and BC = 16 cm. Point P moves from B to C on side BC at a speed of 2 cm per second. How many seconds after point P starts from B does the area of triangle ABC become 35 cm²?

Solution: Let x be the amount of time it takes for the area of triangle ABC to become 35 cm².

In x seconds the length of BP is \(2x\) (cm).

\[
\text{(Area of triangle ABC)} = \frac{1}{2} \times \text{BP} \times \text{AB}, \quad \frac{1}{2} \times 2x \times 7 = 35
\]

Solving this, you get \(x = 5\). Therefore, the area of triangle ABC becomes 35 cm² in 5 seconds.

Answer: In 5 seconds

16 In triangle ABC on the right, angle B is a right angle, AB = 12 cm, and BC = 27 cm. Point P moves from B to C on side BC at a speed of 3 cm per second. How many seconds after point P starts from B does the area of triangle ABC become 108 cm²?

17 In rectangle ABCD on the right, AB = 6 cm and BC = 10 cm. Point P moves from D to A on side DA at a speed of 2 cm per second. Answer the following questions.

(1) Write the simplest expression in terms of \(x\) to indicate the length of AP \(x\) seconds after point P starts from vertex D.

(2) How many seconds after point P starts from vertex D does the area of triangle ABP become 12 cm²?

18 Answer the following questions.

(1) In trapezoid ABCD on the right, angle A and angle B are right angles, AD = 16 cm, BC = 24 cm, and AB = 15 cm. Point P moves from A to B on side AB at a speed of 3 cm per second. How many seconds after point P starts from vertex A does the area of triangle APD become equal to that of triangle BPC?

(2) In rectangle ABCD on the right, AB = 18 cm and BC = 30 cm. Point P moves from D to A on side DA at a speed of 3 cm per second, and point Q moves from B to C on side BC at 2 cm per second. If P and Q start from vertex D and vertex B respectively at the same time, in how many seconds does the area of trapezoid PBQC become 270 cm²?
Chapter 3  Equations

Let's learn the basics  Problems about regularities

Question  As shown in the figure on the right, you can make squares by arranging matchsticks from left to right. Answer the following questions.

(1) How many matchsticks do you need to make \( n \) squares? Write an expression in terms of \( n \).

(2) How many squares can you make with 58 matchsticks?

Solution  (1) As shown in the figure on the right, starting with one matchstick, you can make one square each time you add three matchsticks. So the number of matchsticks you need to make \( n \) squares is 

\[
1 + 3n = 3n + 1 \text{ (matchsticks)}
\]

(2) Since \( 3n + 1 = 58 \), \( n = 19 \).

Answer  (1) \( 3n + 1 \) (matchsticks)  (2) 19 squares

19  As shown in the figure on the right, you can make equilateral pentagons by arranging matchsticks from left to right. Answer the following questions.

□(1) How many matchsticks do you need to make \( n \) equilateral pentagons? Write an expression in terms of \( n \).

□(2) How many equilateral pentagons can you make with 93 matchsticks?

20  The figure on the right shows natural numbers arranged regularly. Answer the following questions.

□(1) When the sum of three numbers enclosed with [ ] as shown in the figure is 110, what is the smallest number of the three?

□(2) Enclose three numbers arranged vertically with \( \begin{array}{c} a \\ b \\ c \end{array} \) as shown in the figure and add them up. When the sum is 156, what is the greatest number of the three?

21  As shown in the figure on the right, some marbles are arranged like a pyramid. There is one marble on the highest tier, and the number of marbles increases one by one as you go down the tiers. When the sum of the number of marbles on three consecutive tiers is 84, from what number tier to what number tier are they?
Answer the following questions.

1) A boy started running from point A to point B. Three minutes later his brother started running to catch up with him. The boy ran at a speed of 120 m per minute, and his brother ran at 150 m per minute. How many minutes after his brother started did he catch up with the boy?

2) You walked 12 km on the road over a hill. It took you 3 hours and 20 minutes to walk up the hill at a speed of 3 km per hour and down at 5 km per hour. What was the distance between the starting point and the hilltop?

Answer the following questions.

1) You fixed the tag price of an article, expecting a profit of 20% of its cost price. You sold it at a discount of 300 yen. The profit was 12% of the cost price. What was the cost price of this article?

2) Suppose you mix 2% salt water and 22% salt water to make 300 g of 7% salt water. How many grams each of 2% and 22% salt water do you need?

Answer the following questions.

1) Last year there were 300 boy and girl students in a school. This year the number of boys has decreased by 10% and the number of girls has increased by 5%, and there are 2% fewer students than last year. How many boys and how many girls are there in this school this year?

2) There is a cuboid with a length of 4 cm and a width of 10 cm. Its height is greater than its width, and its volume is 520 cm³. How many centimeters greater is the height than the width?

Answer the following questions.

1) In rectangle ABCD on the right, AD = 14 cm. Point P moves from A to D and back on side AD. Point Q moves from B to C and back on side BC. They start at the same time. The speed of point P and point Q are 3 cm and 4 cm per second, respectively. How many seconds after they start do the length of AP and the length of BQ first become equal?

2) As shown in the figure on the right, some flowerpots are arranged in a line at regular intervals. When you put planters with flowers around each flowerpot from left to right, how many flowerpots can you enclose with 296 planters?
1. **How to solve equations**
   Solve the following equations.
   - $(1) \ x + 7 = 3$
   - $(2) \ -4x = 28$
   - $(3) \ \frac{3}{5}x = 6$
   - $(4) \ 15 - 4x = -5$
   - $(5) \ 8x - 56 = x$
   - $(6) \ 5x + 6 = 4x + 2$
   - $(7) \ 3x + 6 = 5x - 6$
   - $(8) \ 5x - 12 = 6x - 9$
   - $(9) \ -6x + 5 = -8x + 17$

2. **How to solve equations containing parentheses**
   Solve the following equations.
   - $(1) \ 3(x + 4) = 5x - 6$
   - $(2) \ 4x - 8(x + 2) = 0$
   - $(3) \ 5(x - 2) - (2x - 7) = 9$
   - $(4) \ 2(x - 5) = 3(2 - x) + 14$

3. **How to solve equations containing decimals or fractions, How to solve equations using proportional expressions**
   Solve the following equations.
   - $(1) \ 2.6x - 0.8 = 3.3x + 1.3$
   - $(2) \ 0.17x - 0.6 = 0.08(x - 3)$
   - $(3) \ \frac{1}{2}x - 1 = \frac{2}{3}x$
   - $(4) \ \frac{x - 3}{2} + \frac{x + 1}{3} = 3$
   - $(5) \ x : 12 = \frac{8}{3}$
   - $(6) \ \frac{5}{2} \div \frac{4}{x - 6}$

4. **Using equations**
   Answer the following questions.
   - $(1) \ When \ the \ solution \ to \ the \ equation \ in \ terms \ of \ x, \ \frac{3x - a}{2} = a - x, \ is \ x = -3, \ find \ the \ value \ for \ a.$
   - $(2) \ You \ paid \ 920 \ yen \ for \ 12 \ pencils \ priced \ at \ either \ 70 \ yen \ or \ 90 \ yen. \ How \ many \ 70-\ yen \ pencils \ and \ how \ many \ 90-\ yen \ pencils \ did \ you \ buy?$
   - $(3) \ Suppose \ you \ make \ salt \ water \ whose \ concentration \ is \ 6\%. \ If \ you \ mix \ 800 \ g \ of \ 5\% \ salt \ water \ and \ a \ certain \ amount \ of \ 8\% \ salt \ water, \ how \ many \ grams \ of \ 8\% \ salt \ water \ do \ you \ need?$
   - $(4) \ There \ is \ a \ rectangle \ whose \ circumference \ is \ 50 \ cm. \ Its \ length \ is \ 5 \ cm \ longer \ than \ its \ width. \ What \ is \ the \ width \ of \ this \ rectangle?$
1. Answer the following questions.

(1) There is a three-figure natural number whose tens digit is 2. The sum of its hundreds digit and ones digit is 14. If you exchange the hundreds digit and ones digit, the new number is 198 greater than the original number. Find the original number.

(2) Suppose you distribute some tangerines equally to some students. If you give each student 10 tangerines, 5 students have no tangerine and you have only 6 tangerines left. If you give them 7 tangerines, every student can get 7 and you have 10 left over. How many students and how many tangerines are there?

(3) You checked the long jump records of 120 boy and girl students. The boys’ average was 3.7 m, the girls’ average was 3.1 m, and the overall average was 3.42 m. Find the number of boys.

(4) A ball-point pen is 30 yen more expensive than a pencil. You had just enough money to buy 5 ball-point pens and 5 pencils. As all items were sold at a 10% discount, you were able to get 6 ball-point pens and 5 pencils. How much is a pencil without a discount?

2. A and B run together every morning on a circular jogging trail whose circumference is 1.2 km. One morning, B started first. Two minutes later, A started at the same point and ran in the same direction. B ran at a speed of 150 m per minute, and A ran at 180 m per minute. Answer the following questions.

(1) Write the simplest expressions in terms of $x$ to indicate A’s travel distance and B’s travel distance $x$ minutes after B started.

(2) How many minutes after B started did A catch up with B for the first time?

(3) How many minutes after B started did A catch up with B again after overtaking B?

3. You can arrange white squares and black squares regularly to make the figures shown on the right. Answer the following questions.

(1) Write the simplest expression in terms of $n$ to indicate the number of white squares you need to make an $n$-tier figure.

(2) In order to make a figure using 20 white squares, how many black squares do you need?
Complements

Properties of inequalities

Property 1 If $A < B$, then $A + C < B + C$, $A - C < B - C$.
(If the same number is added to both sides or subtracted from both sides of an inequality, the order of the resulting inequality is unchanged.)

Property 2 If $A < B$, $C > 0$, then $AC < BC$, $\frac{A}{C} < \frac{B}{C}$.
(If both sides of an inequality are multiplied or divided by the same positive number, the order of the resulting inequality is unchanged.)

Property 3 If $A < B$, $C < 0$, then $AC > BC$, $\frac{A}{C} > \frac{B}{C}$.
(If both sides of an inequality are multiplied or divided by the same negative number, the order of the resulting inequality is reversed.)

1 When $a < b$, fill in the blank with a suitable inequality sign.

□ (1) $a + 3 \square b + 3$
□ (2) $a - 8 \square b - 8$
□ (3) $0.2a \square 0.2b$
□ (4) $-12a \square -12b$
□ (5) $a \square b$
□ (6) $-\frac{a}{3} \square -\frac{b}{3}$

2 Solving inequalities using their properties

Values that work for the letter in an inequality are called the solutions to the inequality.

Finding the solutions is called solving the inequality.

When expressing solutions to an inequality on the number line, either "○" or "●" is used. ○ indicates that $a$ is not included in the solutions ($x > a$, $x < a$), while ● indicates that $a$ is included in the solutions ($x \geq a$, $x \leq a$).

- Solutions to an inequality expressed on the number line

1 $x > a$
2 $x \geq a$
3 $x < a$
4 $x \leq a$

(1) $x + 9 > 6$
   Property 1
   $x > -3$
   $x + 9 - 9 > 6 - 9$
   $x > -3$
   $-5 - 4 - 3 - 2 - 1 0$

(2) $7x \leq 28$
   Property 2
   $\frac{7x}{7} \leq \frac{28}{7}$
   $x \leq 4$
   $0 1 2 3 4 5$

(3) $-\frac{x}{4} > 3$
   Property 3
   $x \times (-4) < 3 \times (-4)$
   $x < -12$
   $-15 - 14 - 13 - 12 - 11 - 10$

2 Solve the following inequalities and express where the solution set exists on the number line.

□ (1) $x + 2 < 6$
□ (2) $x + 7 \geq 4$
□ (3) $-4x > -20$
□ (4) $2x - 1 \geq -5$
□ (5) $-2x - 9 < 7$
□ (6) $5x + 24 \leq 7x$
Solving inequalities by transposing terms

You can solve an inequality by transposing terms so that there are only letter terms on the left side and numbers on the right side, as you do when solving a linear equation. Note that the order of the resulting inequality is reversed if you multiply or divide both sides of an inequality by the same negative number.

(1) $4x - 3 \leq 2x - 9$
   
   $4x - 2x \leq -9 + 3$
   
   $2x \leq -6$
   
   $x \leq -3$

(2) $3x + 13 > 8x + 3$
   
   $3x - 8x > 3 - 13$
   
   $-5x > -10$
   
   $x < 2$

The order of the resulting inequality is reversed.

3 Solve the following inequalities.

- (1) $5x - 1 \geq -7 - x$
- (2) $6x + 7 > 3x - 8$
- (3) $8x + 3 < 3x - 12$
- (4) $x + 17 \leq 7x + 5$
- (5) $3x - 29 > 7x + 3$
- (6) $4x - 2 \geq 1 - 5x$
- (7) $8x - 3 \leq 9x + 4$
- (8) $15 - 6x > 20 - 11x$
- (9) $9x + 3 \geq 13x + 19$

4 Solve the following inequalities.

- (1) $5(x - 6) < 3x$
- (2) $7x - 1 \leq 3(2x - 1)$
- (3) $2(x - 5) > 4(2x - 4)$
- (4) $0.7x + 0.4 > 0.2x + 1.9$
- (5) $\frac{1}{2}x > \frac{x + 3}{8}$
- (6) $\frac{3}{5}x - \frac{1}{5} \geq \frac{1}{4}x + \frac{1}{2}$

4 Finding integers

**Question:** Find the greatest integer of the solutions to the inequality $6x - 1 > 8x - 9$.

**Solution:** Solving the inequality, you get $x < 4$. So you can express where the solution set exists on the number line as shown in the figure on the right.

Therefore, the greatest integer of the solutions to this inequality is 3.

**Answer:** 3

5 Answer the following questions.

- (1) Find the greatest integer of the solutions to the inequality $3x - 8 > 7x + 4$.
- (2) Find the smallest integer of the solutions to the inequality $2x + 9 \leq 5x - 1$.
- (3) How many natural numbers are there in the solution set for the inequality $2(-3x + 1) > 9(x - 3) - 11$?
- (4) Find the smallest integer of the solutions to the inequality $1.3x - 4 \geq \frac{4x - 7}{5}$.

**Point:**

1. When solving an inequality that contains coefficients in decimal or fraction form, you should first change the coefficients into integers by multiplying both sides by powers of ten or the least common multiple of the denominators, as you do when solving a linear equation.
1. **Functions, Variables and domains**
   - (1) Letters that can take on a variety of values are called variables.
   - (2) Suppose there are two variables $x$ and $y$ that change together. When you determine the value of $x$, if there is a unique corresponding value of $y$, $y$ is a function of $x$.
   - (3) When a variable is within a range of values, that range is called the domain of the variable.

2. **Proportion and inverse proportion**
   - (1) When $y$ is a function of $x$ and the relationship between them is expressed as $y=ax$ ($a \neq 0$), $y$ is proportional to $x$ and $a$ is called the constant of proportion.
   - (2) When $y$ is a function of $x$ and the relationship between them is expressed as $y=\frac{a}{x}$ ($a \neq 0$), $y$ is inversely proportional to $x$ and $a$ is called the constant of proportion.

3. **Coordinates**
   - (1) Imagine two number lines intersecting perpendicularly at point O, which is their common origin, as in the figure on the right. In this figure, the horizontal number line is called the $x$-axis, and the vertical number line is called the $y$-axis. Both $x$-axis and $y$-axis are collectively called the coordinate axes, and point O is called the origin.
   - (2) Point P in the figure on the right corresponds to a pair of values $x=3$ and $y=2$. They are expressed as $(3, 2)$ and called the coordinates of point P. 3 is the $x$-coordinate and 2 is the $y$-coordinate of point P.
   - (3) The coordinates of the reflection of point P $(a, b)$ about the $x$-axis are $(a, -b)$, those of the reflection of point P about the $y$-axis are $(-a, b)$, and those of the reflection of point P about the origin are $(-a, -b)$.
   - (4) The coordinates of the midpoint between P $(a, b)$ and Q $(c, d)$ are $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

4. **Graphing proportion and inverse proportion**
   - (1) The graph of the proportional relationship
     
     
     $y=ax$ is a straight line that passes through the origin.

   - (2) The graph of the inversely proportional relationship
     
     
     $y=\frac{a}{x}$ is two curves called a hyperbola.
1) Proportion and inverse proportion

Let's learn the basics

1. Letters that can take on a variety of values are called variables.
2. Suppose there are two variables \(x\) and \(y\) that change together. When you determine the value of \(x\), if there is a unique corresponding value of \(y\), \(y\) is a function of \(x\).

Question: Suppose you cut \(x\) pieces of 8-cm ribbon from a 1-m ribbon. Letting \(y\) be the length of the piece that is left uncut, answer the following questions.

1. Express \(y\) in terms of \(x\).
2. Is \(y\) a function of \(x\)?

Solution: (1) (The length of the piece that is left uncut) = (The original length) - (The sum of the lengths of the pieces that were cut off), so
\[
y = 100 - 8x \
\]
When expressing \(y\) in terms of \(x\), the term containing a letter is usually placed in front of the term that is just a number. For example, \(y = -8x + 100\).
(2) When you determine the value of \(x\), there is a unique corresponding value of \(y\). Therefore, \(y\) is a function of \(x\).

Answer: (1) \(y = -8x + 100\) (2) Yes

1) Suppose it took \(x\) hours to walk \(y\) km at a speed of 5 km per hour. Answer the following questions.

1. Express \(y\) in terms of \(x\).
2. Is \(y\) a function of \(x\)?

2) Choose all options indicating that \(y\) is a function of \(x\) and answer using numbers (i) to (iv).

(i) The circumference of a square with a side of \(x\) cm is \(y\) cm.
(ii) A person's height is \(x\) cm and his weight is \(y\) kg.
(iii) You traveled \(y\) km by taxi and paid a fare of \(x\) yen.
(iv) It took \(y\) minutes to empty a tank filled with 50 L of water by draining \(x\) L per minute.

3) Suppose a 30-cm candle burns to become 2 cm shorter per minute. Letting \(y\) be the length of the candle \(x\) minutes after it was lit, answer the following questions.

1. Express \(y\) in terms of \(x\).

2. Fill in the blanks of the following table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Find the value for \(y\) when \(x = 13\).
4. Find the value for \(x\) when \(y = 8\).
Let's learn the basics  Variables and domains

When a variable is within a range of values, that range is called the domain of the variable.

[Example] Suppose you pour water at a rate of 5 L per minute into a 30-L tank until it is filled up. Let \( y \) be the amount of water in the tank \( x \) minutes after you start pouring in the water. Since the tank is filled up in \( 6 \) (=30÷5) minutes, the value of \( x \) is at least 0 and at most 6. Therefore the domain of \( x \) can be expressed as

\[
0 \leq x \leq 6
\]

How to express a variety of domains

(1) The domain of a variable \( x \) is more than 0 and less than 6. \( 0 < x < 6 \)

(2) The domain of a variable \( x \) is more than 0 and at most 6. \( 0 < x \leq 6 \)

(3) The domain of a variable \( x \) is at least 0 and less than 6. \( 0 \leq x < 6 \)

\( x > 0 \) indicates that \( x \) is a positive number, while \( x < 0 \) indicates that \( x \) is a negative number.

4. Express the domain of the following variables using inequality signs.

- (1) The domain of a variable \( x \) is at least -1 and at most 5.
- (2) The domain of a variable \( y \) is more than -5 and less than -2.
- (3) The domain of a variable \( x \) is at least 7 and less than 12.
- (4) The domain of a variable \( y \) is more than -3 and at most 9.

5. Find the domain of \( x \) and the domain of \( y \) in the following.

- (1) Suppose you pour water at a rate of 6 L per minute into an empty 120-L tank until it is filled up. The amount of water in the tank is \( y \) L, \( x \) minutes after you start pouring in the water.

- (2) Suppose you go to a place 60 km away by bicycle. You ride at a speed of 12 km per hour. \( x \) hours after you start, you are \( y \) km away from the destination.

6. In rectangle ABCD on the right, \( AB = 8 \) cm and \( AD = 12 \) cm.

Point \( P \) moves from \( B \) to \( C \) on side BC. When point \( P \) is \( x \) cm away from \( B \), the area of triangle ABP is \( y \) cm². Answer the following questions.

- (1) Express \( y \) in terms of \( x \).

- (2) Find the domain of \( x \) and the domain of \( y \).
When the relationship between \( x \) and \( y \) is expressed as \( y=ax \) (\( a \) is a constant that is not 0), \( y \) is proportional to \( x \) and \( a \) is called the constant of proportion. When \( y \) is proportional to \( x \) and \( x \neq 0 \), the value of \( \frac{y}{x} \) is fixed and equal to the constant of proportion.

Suppose you walk at a speed of 70 m per minute. Letting \( y \) be the distance you travel in \( x \) minutes, the relationship between \( x \) and \( y \) is expressed in the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>70</td>
<td>140</td>
<td>210</td>
<td>280</td>
<td>350</td>
<td>420</td>
<td>490</td>
<td>560</td>
</tr>
</tbody>
</table>

The table above indicates the following.

1. As the value of \( x \) is multiplied 2 times, 3 times, 4 times, etc., the value of \( y \) is also multiplied 2 times, 3 times, 4 times, etc.
2. The value of \( y \) is 70 times the value of \( x \). Therefore, \( y \) can be expressed in terms of \( x \) as \( y=70x \).

Fill in the blanks of the table above.

Write an expression to indicate the following relationship between \( x \) and \( y \). Then, if \( y \) is proportional to \( x \), write the constant of proportion. If it is not, mark a cross (\( \times \)).

1. When walking for \( x \) minutes at 55 m per minute, the travel distance is \( y \) m.

2. The area of a square with a side of \( x \) cm is \( y \) cm².

3. Suppose you buy some ribbon whose price is 120 yen per meter. You have to pay \( y \) yen to buy \( x \) m of this ribbon.

4. A 50-g box filled with \( x \) 180-g balls weighs \( y \) g in total.

5. The long hand of a clock turns \( y \) degrees in \( x \) minutes.

When the relationship between \( x \) and \( y \) is expressed as \( y=3x \), answer the following questions.

1. What is the constant of proportion?

2. When the value of \( x \) is the following, find the value for \( y \).
   - \( x=2 \)
   - \( x=-5 \)
   - \( x=-9 \)
   - \( x=\frac{5}{6} \)

3. When the value of \( y \) is the following, find the value for \( x \).
   - \( y=-3 \)
   - \( y=12 \)
   - \( y=39 \)
   - \( y=-7 \)
When \( y \) is proportional to \( x \), the relationship between \( x \) and \( y \) is expressed as \( y = ax \). You can find the value for \( a \) by substituting the given values for \( x \) and \( y \).

**Question**

1. **Express \( y \) in terms of \( x \).**
2. **When \( x = -5 \), find the value for \( y \). When \( y = 45 \), find the value for \( x \).**

**Solution**

1. Since \( y \) is proportional to \( x \), their relationship is expressed as \( y = ax \), letting \( a \) be the constant of proportion. Substituting 3 and 15 for \( x \) and \( y \) respectively gives you \( 15 = a \times 3 \), so \( a = 5 \).
2. Substituting \(-5\) for \( x \) in the expression \( y = 5x \) gives you \( y = 5 \times (-5) = -25 \).
3. Substituting 45 for \( y \) in the expression \( y = 5x \) gives you \( 45 = 5x \), so \( x = 9 \).

**Answer**

1. \( y = 5x \)  
2. When \( x = -5 \), \( y = -25 \). When \( y = 45 \), \( x = 9 \).

9

When \( y \) is proportional to \( x \), answer the following questions.

- **1) When \( x = 3 \), \( y = 6 \). Express \( y \) in terms of \( x \).**
- **2) When \( x = 2 \), \( y = -14 \). Express \( y \) in terms of \( x \).**
- **3) When \( x = -2 \), \( y = 6 \). Express \( y \) in terms of \( x \).**
- **4) When \( x = -3 \), \( y = -12 \). Express \( y \) in terms of \( x \).**
- **5) When \( x = 10 \), \( y = -5 \). Express \( y \) in terms of \( x \).**

10

When \( y \) is proportional to \( x \), answer the following questions.

- **1) When \( x = 4 \), \( y = -8 \). When \( x = 6 \), find the value for \( y \).**
- **2) When \( x = 6 \), \( y = 24 \). When \( x = 3 \), find the value for \( y \).**
- **3) When \( x = -8 \), \( y = -16 \). When \( x = 10 \), find the value for \( y \).**
- **4) When \( x = -3 \), \( y = -9 \). When \( y = 18 \), find the value for \( x \).**
- **5) When \( x = 2 \), \( y = -8 \). When \( y = -10 \), find the value for \( x \).**

11

Answer the following questions.

- **1) A car consumes 5 L of gasoline to run 80 km. Suppose that it consumes \( x \) L of gasoline to run \( y \) km.**
  - **1. Express \( y \) in terms of \( x \).**
  - **2. How many km does this car run when it consumes 6.5 L of gasoline?**
  - **3. How many L of gasoline does this car need to run 400 km?**

- **2) A piece of wire weighs 144 g when it is 3 m long. The same wire weighs \( y \) g when it is \( x \) m long. Express \( y \) in terms of \( x \), and find the length of this wire when it weighs 264 g.**
Let's learn the basics 5 Inverse proportion

When the relationship between $x$ and $y$ is expressed as $y = \frac{a}{x}$ (where $a$ is a constant that is not 0), $y$ is inversely proportional to $x$ and $a$ is called the constant of proportion. When $y$ is inversely proportional to $x$, the value of $xy$ is fixed and equal to the constant of proportion.

If you pour water at a rate of $x$ L per minute into a 420-L tank, you can fill it up in $y$ minutes. The relationship between $x$ and $y$ is expressed in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>420</td>
<td>210</td>
<td>140</td>
<td>105</td>
<td>84</td>
<td>60</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table above indicates the following.

(1) As the value of $x$ is multiplied 2 times, 3 times, 4 times etc., the value of $y$ is also multiplied $\frac{1}{2}$ times, $\frac{1}{3}$ times, $\frac{1}{4}$ times, etc.

(2) The product $xy$ of corresponding $x$ and $y$ values is always 420. Therefore, $y$ can be expressed in terms of $x$ as $y = \frac{420}{x}$.

2 Fill in the blanks of the table above.

12 Write an expression to indicate the following relationship between $x$ and $y$. Then, if $y$ is inversely proportional to $x$, write the constant of proportion; if it isn’t, mark a cross ($\times$).

☐ (1) The area of a triangle with an $x$-cm base and a $y$-cm height is 15 cm².

☐ (2) The circumference of a rectangle of $x$ cm long and $y$ cm wide is 20 cm.

☐ (3) It takes $y$ minutes to walk 800 m at a speed of $x$ m per minute.

☐ (4) The length of the daytime and the nighttime of a day is $x$ hours and $y$ hours, respectively.

☐ (5) Suppose you pour water into a container whose inner size is 10 cm by 8 cm by 6 cm. If you pour water into the container at a rate of $x$ cm³ per second, you can fill it up in $y$ seconds.

13 When the relationship between $x$ and $y$ is expressed as $y = \frac{180}{x}$, answer the following questions.

☐ (1) What is the constant of proportion?

☐ (2) When the value of $x$ is the following, find the value for $y$.

   ① $x = 3$  ② $x = -10$  ③ $x = -36$  ④ $x = 24$

☐ (3) When the value of $y$ is the following, find the value for $x$.

   ① $y = -2$  ② $y = 6$  ③ $y = -15$  ④ $y = \frac{25}{3}$
When $y$ is inversely proportional to $x$, their relationship is expressed as $y = \frac{a}{x}$. Letting $a$ be the constant of proportion, substituting 4 and 9 for $x$ and $y$ respectively gives you $9 = \frac{a}{4}$, so $a = 36$.

Therefore the right expression is $y = \frac{36}{x}$.

1. Substituting 12 for $x$ in the expression $y = \frac{36}{x}$ gives you $y = \frac{36}{12} = 3$.

To find the value for $x$, you should first change the expression $y = \frac{36}{x}$ to $xy = 36$. Substituting $-18$ for $y$ in this expression gives you $x \times (-18) = 36$, so $x = -2$.

Answer: (1) $y = \frac{36}{x}$
(2) When $x=12$, $y=3$. When $y=-18$, $x=-2$.

14 When $y$ is inversely proportional to $x$, answer the following questions.

☐ (1) When $x=3$, $y=1$. Express $y$ in terms of $x$.
☐ (2) When $x=2$, $y=-5$. Express $y$ in terms of $x$.
☐ (3) When $x=-7$, $y=2$. Express $y$ in terms of $x$.
☐ (4) When $x=-5$, $y=-3$. Express $y$ in terms of $x$.
☐ (5) When $x=-6$, $y=\frac{4}{3}$. Express $y$ in terms of $x$.

15 When $y$ is inversely proportional to $x$, answer the following questions.

☐ (1) When $x=2$, $y=6$. When $x=3$, find the value for $y$.
☐ (2) When $x=3$, $y=-8$. When $x=-4$, find the value for $y$.
☐ (3) When $x=-4$, $y=3$. When $x=-24$, find the value for $y$.
☐ (4) When $x=-6$, $y=-3$. When $y=9$, find the value for $x$.
☐ (5) When $x=1$, $y=-4$. When $y=12$, find the value for $x$.

16 Gear A has 30 teeth and rotates 120 times per minute. Gear B meshes with Gear A. Answer the following questions.

☐ (1) Supposing Gear B has $x$ teeth and rotates $y$ times per minute, express $y$ in terms of $x$.
☐ (2) When Gear B has 45 teeth, how many times per minute does it rotate?
☐ (3) To make Gear B rotate 50 times per minute, how many teeth does it need to have?
Exercises

1 Write an expression to indicate the following relationship between \( x \) and \( y \). If \( y \) is proportional to \( x \), mark a circle (○). If \( y \) is inversely proportional to \( x \), mark a triangle (△). If \( y \) is neither proportional nor inversely proportional to \( x \), mark a cross (×).

(1) The price of a 5-m ribbon is 1000 yen. You need to pay \( y \) yen to buy \( x \) m of this ribbon.

(2) You have 500 yen. If you buy something and pay \( x \) yen, you have \( y \) yen left.

(3) If you cut a 2-m string into \( x \) pieces so that each piece is the same length, each piece is \( y \) cm long.

2 Answer the following questions.

(1) Write an expression to indicate the relationship between \( x \) and \( y \) shown in the following table, then fill in the blanks.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>30</td>
<td>45</td>
<td>60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>-1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-2</td>
<td>9</td>
<td>3</td>
</tr>
</tbody>
</table>

(2) \( y \) is proportional to \( x \). When \( x = 6 \), \( y = -10 \). Find the value for \( y \) when \( x = -15 \).

(3) \( y \) is inversely proportional to \( x \). When \( x = -\frac{3}{2} \), \( y = -\frac{4}{3} \). Find the value for \( x \) when \( y = -6 \).

3 There is a 1.5-m stick standing vertically, casting a 0.9-m shadow on the ground. Letting \( y \) be the length of the shadow of an \( x \)-m stick at the same time, answer the following questions.

(1) Express \( y \) in terms of \( x \).

(2) At the same place and at the same time, a tree casts a 7.2-m shadow on the ground. How many meters tall is the tree?

4 A lever is balancing as shown in the figure on the right. The weight (\( y \) g) of a weight is inversely proportional to the distance (\( x \) cm) from the fulcrum. Answer the following questions.

(1) Express \( y \) in terms of \( x \).

(2) When \( y = 25 \), find the value for \( x \).
Let's learn the basics 1 Points and coordinates

When point P corresponds to a pair of $x$ and $y$ values, where $x = a$ and $y = b$, the coordinates of point P are expressed as $(a, b)$. This point is also expressed as $P(a, b)$.

[Example] ① In the figure on the right, point A is 4 spaces to the left of the origin (in the negative direction on the $x$-axis) and 3 spaces above the origin (in the positive direction on the $y$-axis). It corresponds to $x = -4$, $y = 3$, so the coordinates of point A are $(-4, 3)$.

② Likewise, the coordinates of point B, C, D, and E are:

$B(3, 2), C(-2, 5), D(-4, -2), E(4, -4)$.

1 State the coordinates of the following points in the figure on the right.

[Diagram]

- A
- B
- C
- D
- E
- F
- G
- H

2 Plot the points represented by the following coordinates in the figure on the right.

- A(5, 2)
- B(-3, 5)
- C(0, -3)
- D(4, 4)
- E(1, 6)
- F(3, -4)
- G(-4, 0)
- H(-6, -5)

3 What are the signs of the $x$-coordinate and $y$-coordinate of the following point? Express them using + and - signs, such as (+, -).

[Diagram]

- A point below the $x$-axis and to the right of the $y$-axis
- A point above the $x$-axis and to the left of the $y$-axis
- A point below the $x$-axis and to the left of the $y$-axis
- A point above the $x$-axis and to the right of the $y$-axis

Point

① Imagine two number lines intersecting perpendicularly at point O, which is their common origin. In this figure, the horizontal number line is called the $x$-axis and the vertical number line is called the $y$-axis. Both $x$-axis and $y$-axis are collectively called the coordinate axes, and point O is called the origin.

② Coordinates are always expressed as $(x$-coordinate, $y$-coordinate).
Let's learn the basics 2 Reflection of a point, Midpoint, Moving a point

1. The coordinates of the reflection of \( P(a, b) \) about the \( x \)-axis are \( (a, -b) \).
   The coordinates of the reflection of \( P(a, b) \) about the \( y \)-axis are \( (-a, b) \).
   The coordinates of the reflection of \( P(a, b) \) about the origin are \( (-a, -b) \).

2. The coordinates of the midpoint between \( P(a, b) \) and \( Q(c, d) \) are \( \left( \frac{a+c}{2}, \frac{b+d}{2} \right) \).

**Question**
State the coordinates of the following point in the figure on the right.

1. The reflection of \( A \) about the \( x \)-axis
2. The reflection of \( A \) about the \( y \)-axis
3. The reflection of \( A \) about the origin
4. The midpoint between \( A \) and \( B \)
5. The point 3 spaces to the right and 6 spaces down from \( A \)

**Solution**
(1) Changing the sign of the \( y \)-coordinate of \( A(3, 2) \) gives you \( (3, -2) \).
(2) Changing the sign of the \( x \)-coordinate of \( A(3, 2) \) gives you \( (-3, 2) \).
(3) Changing both signs of the \( x \)-coordinate and \( y \)-coordinate of \( A(3, 2) \) gives you \( (-3, -2) \).
(4) The coordinates of the midpoint between \( A(3, 2) \) and \( B(-5, 6) \) are \( (-1, 4) \), since the \( x \)-coordinate is \( \frac{3+(-5)}{2} = -1 \), and the \( y \)-coordinate is \( \frac{2+6}{2} = 4 \).
(5) Moving 3 spaces to the right, the \( x \)-coordinate becomes \( 3+3=6 \).
Moving 6 spaces down, the \( y \)-coordinate becomes \( 2+(-6) = -4 \). Therefore, the coordinates of the point move to become \( (6, -4) \). 

**Answer**
(1) \( (3, -2) \)  (2) \( (-3, 2) \)  (3) \( (-3, -2) \)  (4) \( (-1, 4) \)  (5) \( (6, -4) \)

4. Find the coordinates of the point that is the reflection of the following point about the \( x \)-axis, about the \( y \)-axis, and about the origin.
   - \( A(3, 4) \)
   - \( B(-2, 3) \)
   - \( C(4, -5) \)
   - \( D(-3, -6) \)
   - \( E(8, 8) \)
   - \( F(-1, 1) \)

5. Find the coordinates of the midpoint between the following pair of points.
   - \( A(2, 5), B(4, 5) \)
   - \( C(3, -5), D(3, 3) \)
   - \( E(-8, -2), F(-4, 6) \)
   - \( G(5, 2), H(-9, -8) \)

6. Find the coordinates of point \( P(2, 5) \) after it moves as follows.
   - \( A(1) \) 3 spaces to the right
   - \( A(2) \) 6 spaces to the left
   - \( A(3) \) 2 spaces up
   - \( A(4) \) 7 spaces down
   - \( A(5) \) 1 space to the right and 5 spaces down
   - \( A(6) \) 10 spaces to the left and 10 spaces up
Chapter 4  Proportion and inverse proportion

Let's learn the basics  3  Graphing proportion

The graph of the proportional relationship \( y=ax \) (\( a \) is a constant that is not 0) is a straight line that passes through the origin.

**Question**  Graph \( y=2x \).

**Solution**  The tables below shows the corresponding \( x \) and \( y \) values in the proportional relationship \( y=2x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -3 )</th>
<th>( -2 )</th>
<th>( -1 )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 2 )</th>
<th>( 3 )</th>
<th>( ... )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( -6 )</td>
<td>( -4 )</td>
<td>( -2 )</td>
<td>( 0 )</td>
<td>( 2 )</td>
<td>( 4 )</td>
<td>( 6 )</td>
<td>( ... )</td>
</tr>
</tbody>
</table>

Using the pairs of \( x \) and \( y \) values in the table above, you can plot the points as shown in the figure on the right. If you plot more points in the same way, the set of these points ends up to be a straight line.

**Answer**  The figure on the right

**Note**  You can also draw the graph by connecting the origin with another point where the relationship \( y=2x \) holds true, since the graph of a proportional relationship is a straight line that passes through the origin.

7  Make tables of the corresponding \( x \) and \( y \) values in the following proportional relationships and graph them.

**Graphs of (1) to (3)**

**Graphs of (4) to (6)**
8 In the following proportional relationship, how does the value of \( y \) change when the value of \( x \) increases by 1?

\( \square 1) \ y = 5x \quad \square 2) \ y = 8x \quad \square 3) \ y = \frac{1}{10}x \)

\( \square 4) \ y = -3x \quad \square 5) \ y = -6x \quad \square 6) \ y = -0.2x \)

9 When the variable \( x \) has a domain shown in the parenthesis, draw the graph of the following proportion and find the domain of \( y \).

\( \square 1) \ y = 2x \quad ( -2 \leq x \leq 2) \)

\( \square 2) \ y = \frac{3}{2}x \quad ( -4 \leq x \leq 2) \)

\( \square 3) \ y = -\frac{1}{2}x \quad ( -6 \leq x \leq 2) \)

\( \square 4) \ y = -\frac{3}{4}x \quad ( -4 \leq x \leq 4) \)

10 Choose the graph representing each of the following proportions \( 1) \) to \( 4) \) and answer using a number.

\( \square 1) \ y = x \quad \square 2) \ y = -2x \)

\( \square 3) \ y = \frac{2}{3}x \quad \square 4) \ y = -\frac{2}{3}x \)

Point

i. The graph of the proportional relationship \( y = ax \) \((a \) is a constant that is not 0) is a straight line whose gradient varies according to the value of \( a \). The line rises up to the right when \( a > 0 \) and falls down to the right when \( a < 0 \).
Let's learn the basics 4 Expressing proportional graphs

**Question** There are two graphs of proportional relationships on the right.

Write the expressions of the graphs ① and ②.

**Solution** The graph of a proportional relationship is a straight line that passes through the origin, expressed as \( y = ax \) (\( a \) is the constant of proportion). You can find the value for \( a \) by substituting for \( y = ax \) the values of the \( x \)-coordinate and the \( y \)-coordinate of one point, except for the origin on the graph.

① Since the graph passes through A(5, 3), substituting \( x = 5 \) and \( y = 3 \) for \( y = ax \) gives you \( 3 = a \times 5 \), so \( a = \frac{3}{5} \). Therefore, the expression of this graph is \( y = \frac{3}{5} x \).

② Since the graph passes through B(3, -2), substituting \( x = 3 \) and \( y = -2 \) for \( y = ax \) gives you \( -2 = a \times 3 \), so \( a = -\frac{2}{3} \). Therefore, the expression of this graph is \( y = -\frac{2}{3} x \).

**Answer** ① \( y = \frac{3}{5} x \) ② \( y = -\frac{2}{3} x \)

11 The figures on the right are the graphs of proportional relationships. Answer the following questions.

① Write the expression of the graph ①.

② Write the expression of the graph ②.

③ Write the expression of the graph ③.

④ Write the expression of the graph ④.

12 The figures on the right are the graphs of proportional relationships. Write the expressions of the following graphs and find the values for the letters.

① the expression of the graph ① and the value for \( m \)

② the expression of the graph ② and the value for \( n \)

③ the expression of the graph ③ and the value for \( p \)

④ the expression of the graph ④ and the value for \( q \)
Let's learn the basics  Graphing inverse proportion

**Question** Graph \( y = \frac{6}{x} \).

**Solution** The tables below show the corresponding \( x \) and \( y \) values in the inversely proportional relationship \( y = \frac{6}{x} \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>...</th>
<th>-6</th>
<th>-4</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>...</td>
<td>-1</td>
<td>-3</td>
<td>-6</td>
<td>-3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>...</td>
</tr>
</tbody>
</table>

Using the pairs of \( x \) and \( y \) values in the table above, you can plot ten points such as (1, 6) and (2, 3) etc. If you plot more points in the same way, the set of these points ends up to form curves as shown in the figure on the right.

**Answer** The figure on the right

**Note** The graph of an inversely proportional relationship is a pair of smooth curves called a hyperbola. They are symmetric about the origin.

13 Make tables of the corresponding \( x \) and \( y \) values in the following inversely proportional relationships and graph them.

- **(1)** \( y = \frac{4}{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>...</th>
<th>-4</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>...</th>
</tr>
</thead>
</table>
| \( y \) | ... | ... | ... | ... | ... | ... | ... | ... | ...

- **(2)** \( y = \frac{-8}{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>...</th>
<th>-4</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>...</th>
</tr>
</thead>
</table>
| \( y \) | ... | ... | ... | ... | ... | ... | ... | ... | ...

- **(3)** \( y = \frac{10}{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>...</th>
<th>-4</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>...</th>
</tr>
</thead>
</table>
| \( y \) | ... | ... | ... | ... | ... | ... | ... | ... | ...

14 Choose the graph representing each of the following inverse proportions (1) to (4) and answer using a number.

- **(1)** \( y = \frac{24}{x} \)
- **(2)** \( y = \frac{-6}{x} \)
- **(3)** \( y = \frac{10}{x} \)
- **(4)** \( y = \frac{-18}{x} \)

**Point**

1. The graph of the inversely proportional relationship \( y = \frac{a}{x} \) (\( a \) is a constant that is not 0) is a hyperbola. It consists of a pair of curves that are symmetric about the origin. They never intersect with the \( x \)-axis or the \( y \)-axis no matter how far they are extended.
Let's learn the basics

Expressing inversely proportional graphs

**Question**
There is a graph of an inversely proportional relationship on the right.

**Solution**
The graph of an inversely proportional relationship is expressed as

\[ y = \frac{a}{x} \]  
(a is the constant of proportion). You can find the value for \( a \)

by substituting for \( y = \frac{a}{x} \) the values of the \( x \)-coordinate and the \( y \)-coordinate

of one point on the graph.

Since the graph passes through \( A(2, 6) \), substituting \( x = 2 \) and \( y = 6 \) for \( y = \frac{a}{x} \) gives you \( 6 = \frac{a}{2} \), so \( a = 12 \).

Therefore, the expression of this graph is \( y = \frac{12}{x} \).

**Answer**
\( y = \frac{12}{x} \)

**15** The figures on the right are the graphs of inversely proportional
relationships. Answer the following questions.

□(1) Write the expression of the graph ①.

□(2) Write the expression of the graph ②.

□(3) Write the expression of the graph ③.

□(4) Write the expression of the graph ④.

**16** The figures ① and ② on the right are the graphs of inversely
proportional relationships. Answer the following questions.

□(1) Write the expressions of the graphs ① and ②.

□(2) Find the values for \( p \) and \( q \).

□(3) Find the coordinates of the point that is the reflection of
the point \( (2, 8) \) about the origin. Is the point on the graph ①?

□(4) On each of the graph ① and the graph ②, find the number of points whose \( x \)-coordinate and
\( y \)-coordinate are both integers.
1. Answer the questions about the figure on the right.
   □(1) Find the coordinates of the points that are the reflections of point A about the x-axis, about the y-axis, and about the origin.
   □(2) Find the coordinates of the midpoint between points B and C.
   □(3) When you move triangle ABC so that point A comes to the origin, find the coordinates of point B.

2. Graph the following expressions.
   □(1) \( y = 4x \)
   □(2) \( y = \frac{4}{3}x \)
   □(3) \( y = -0.8x \)
   □(4) \( y = \frac{8}{x} \)
   □(5) \( y = \frac{24}{x} \)
   □(6) \( y = -\frac{16}{x} \)

3. Answer the questions about the figures below.
   □(1) Express \( y \) in terms of \( x \) for the graphs ① to ④ in Figure 1 and Figure 2.
   □(2) Figure 3 shows a graph of a proportional relationship and a graph of an inversely proportional relationship. Find the values for \( p \), \( q \), and \( r \).
There is a triangle whose area is 100 cm$^2$. Letting $x$ cm be the length of its base and $y$ cm be its height, answer the following questions.

**(1)** Express $y$ in terms of $x$.

**(2)** Graph the relationship between $x$ and $y$.

**(3)** When the length of the base is 25 cm, how many cm is the height?

**(4)** When the height is 10 cm, how many cm is the length of the base?

A piece of wire weighs 60 g for 3 m long. Supposing the same wire weighs $y$ g when it is $x$ m long, answer the following questions.

**(1)** Express $y$ in terms of $x$.

**(2)** Graph the relationship between $x$ and $y$.

**(3)** How many g is this wire when it is 4.5 m long?

A boy and his brother left home at the same time and headed for the station 400 m away. The boy traveled at a speed of 80 m per minute and his brother traveled at 60 m per minute. Supposing they are $y$ m away from their house $x$ minutes after they started, answer the following questions.

**(1)** Express $y$ in terms of $x$ about each of them.

**(2)** Graph the relationship between $x$ and $y$ about each of them.

**(3)** How many minutes after they left home did each of them arrive at the station?

**(4)** How many meters apart were they from each other 2 minutes after they started?
3 Applying coordinates and graphs to figures

Let's learn the basics 1 Coordinates and the area of figures

Question Find the area of triangle ABC having vertices A(-2, 4), B(2, -2), and C(5, 2). Let a space along the coordinate axes be 1 cm.

Solution Make a rectangle ADEF as shown in the figure on the right, and subtract the area of three triangles from the area of this rectangle.

In the rectangle ADEF, AD = 4 - (-2) = 6 (cm)
AP = 5 - (-2) = 7 (cm)
Also, DB = 2 - (-2) = 4 (cm), BE = 5 - 2 = 3 (cm)
CE = 2 - (-2) = 4 (cm), FC = 4 - 2 = 2 (cm)
Therefore the area of triangle ABC is

\[6 \times 7 - \left(\frac{1}{2} \times 4 \times 6 + \frac{1}{2} \times 3 \times 4 + \frac{1}{2} \times 7 \times 2\right) = 17 \text{ (cm}^2)\]

Answer 17 cm²

1 Find the area of the following triangle or quadrilateral, letting a space along the coordinate axes be 1 cm.

[Diagram of triangles and quadrilaterals]

2 Find the area of the following triangle or quadrilateral, letting a space along the coordinate axes be 1 cm.

[Diagram of triangles and quadrilaterals]

- (1) a triangle having vertices A(-4, 3), B(-2, -5), and C(4, -2)
- (2) a triangle having vertices A(4, 5), B(-4, -2), and C(2, 0)
- (3) a quadrilateral having vertices A(3, 4), B(-5, 1), C(-2, -6), and D(4, -1)
Let's learn the basics  Coordinates and parallelogram

**Question** The quadrilateral shown on the right is a parallelogram having vertices A(−3, 2), B(−5, −4), C(2, −1), and D. Find the coordinates of vertex D.

**Solution** When you move B 2 spaces to the right and 6 spaces up, it comes to A. In the same way, moving C 2 spaces to the right and 6 spaces up gives you the position of D. The x-coordinate of D is $2 + 2 = 4$, and its y-coordinate is $-1 + 6 = 5$.

Therefore, the coordinates of D are (4, 5).

**Another solution** Label as M the point where diagonal AC and diagonal BD intersects. M is the midpoint between A and C, so the coordinates of M are $\left(\frac{-3 + 2}{2}, \frac{2 + (-1)}{2}\right)$, that is $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

Next, label the coordinates of D as $(p, q)$. M is also the midpoint between B and D, $\frac{-5 + p}{2} = -\frac{1}{2}$, so $p = 4$. And $\frac{5 + q}{2} = \frac{1}{2}$, so $q = 5$. Therefore, the coordinates of D are (4, 5).

**Answer** (4, 5)

3 Imagine a parallelogram ABCD. When the coordinates of vertices A, B, and C are the following, find the coordinates of vertex D.

- □(1) A(−3, 0), B(−5, −2), C(3, 0)
- □(2) A(−2, 1), B(−3, −5), C(2, −3)
- □(3) A(4, −3), B(1, 3), C(−5, 1)

4 Imagine a parallelogram ABCD. When the coordinates of its three vertices are A(−3, −2), B(1, −3), and C(7, 6), answer the following questions.

- □(1) Label as M the point where the two diagonals intersect. Find the coordinates of M.

- □(2) Find the coordinates of D.

5 Imagine drawing a parallelogram ABCD. When the coordinates of its three vertices are A(−1, 3), B(−5, 0), and C(3, −2), find all the conceivable coordinates of the fourth vertex.

6 There are two points A(3, 4) and B(−3, 2). Imagine a parallelogram ABCD with a side AB. Answer the following questions.

- □(1) When C is the origin, find the coordinates of D.

- □(2) Label as M the point where the two diagonals intersect. When the coordinates of M are (1, −1), find the coordinates of C and those of D.
Let's learn the basics  3  Graphs and figures

Question  Answer the following questions about the figure shown on the right.

(1) Find the value for $a$.
(2) Letting a space along the coordinate axes be 1 cm, find the area of triangle PQR.

Solution  (1) Point P is on the graph $y = 2x$, so its $y$-coordinate is $y = 2 \times 3 = 6$. P(3, 6) is also on the graph $y = \frac{a}{x}$, the value of the constant $a$ is $a = xy = 3 \times 6 = 18$.

(2) Graphs of proportional and inversely proportional relationships are symmetric about the origin, so the coordinates of Q are (-3, -6). The coordinates of R are (-6, -3). As shown in the figure on the right, you can find the area of triangle PQR by subtracting the area of three triangles from the area of a rectangle with a length of $6 - (-6) = 12$ (cm) and a width of $3 - (-6) = 9$ (cm).

\[12 \times 9 - (\frac{1}{2} \times 9 \times 9 + \frac{1}{2} \times 3 \times 3 + \frac{1}{2} \times 6 \times 12) = 27 \text{ (cm}^2)\]

Answer  (1) $a = 18$  (2) 27 cm$^2$

7  Find the following (1) to (3) in the figures (1) and (2) below. (To find (3), let a space along the coordinate axes be 1 cm.)

(1) the value of $a$  (2) the coordinates of Q  (3) the area of triangle PQR

\[\text{(1)}\]

\[\text{(2)}\]

8  In the figure on the right, point P is on the graph of the function $y = 2x$. Label as Q the point where two lines intersect: one line passes through P and is parallel to the $x$-axis, and the other line is the graph of the function $y = \frac{3}{4}x$. Answer the following questions.

\[\text{(1)}\] When the $x$-coordinate of P is 6, find the $y$-coordinate of Q.

\[\text{(2)}\] When the positions of P and Q are shown in (1), find the area of triangle OPQ, letting a space along the coordinate axes be 1 cm.

Point  1. Graphs of proportional and inversely proportional relationships are both symmetric about the origin.
Let's learn the basics

Chapter 4  Proportion and inverse proportion

Let's learn the basics

Graphs and using equations

**Question:** In the figure on the right, point P is on the graph of the function \( y = \frac{1}{2}x \). Imagine two straight lines that pass P: one line is parallel to the \( y \)-axis, and the other line is parallel to the \( x \)-axis. Label as Q and R the points where the two lines intersect with the graph of the function \( y = 2x \). Answer the following questions.

1. When the \( x \)-coordinate of P is 4, find the coordinates of P, Q, and R.
2. When the \( x \)-coordinate of P is 6, find the length of PQ.
3. When the length of PQ is 3, find the coordinates (positive numbers) of P.

**Solution:**

1. The \( x \)-coordinate of P is 4, so its \( y \)-coordinate is \( y = \frac{1}{2} \times 4 = 2 \). Therefore, the coordinates of P are (4, 2).
   The \( x \)-coordinate of Q is also 4, so its \( y \)-coordinate is \( y = 2 \times 4 = 8 \). Therefore the coordinates of Q are (4, 8).
   The \( y \)-coordinate of R is 2, so its \( x \)-coordinate is \( 2 = \frac{1}{2} \times 8 \), then \( x = 1 \). Therefore the coordinates of R are (1, 2).

2. The length of PQ = \( (y \)-coordinate of Q \) - \( (y \)-coordinate of P \) = \( 2 \times 4 - \frac{1}{2} \times 6 \) = 9

3. Letting \( t \) be the \( x \)-coordinate of P, \( 2t - \frac{1}{2} t = 3 \), so \( t = 2 \).

**Answer:**

1. P(4, 2), Q(4, 8), R(1, 2)
2. 9
3. 2

9

In the figure on the right, point P is on the graph of the function \( y = \frac{1}{2}x \). Imagine two straight lines that pass P: one line is parallel to the \( y \)-axis, and the other line is parallel to the \( x \)-axis. Label as Q and R the points where the two lines intersect with the graph of the function \( y = \frac{3}{2}x \). Answer the following questions.

1. When the \( x \)-coordinate of P is 2, find the coordinates of P, Q, and R.
2. When the \( x \)-coordinate of P is 6, find the length of PQ.
3. When the length of PQ is 8, find the coordinates (positive numbers) of P.

10

In the figure on the right, point A is on the graph of the function \( y = \frac{2}{3}x \) and its \( x \)-coordinate is a positive number. The quadrilateral ABCD is a square. Answer the following questions.

1. When the \( x \)-coordinate of B is 6, find the coordinates of A and D.
2. When the length of a side of the square ABCD is 12, find the coordinates of D.
3. When the \( x \)-coordinate of D is 15, find the \( x \)-coordinate of B.
1. Find the area of the following figures, letting a space along the coordinate axes be 1 cm.
   □(1) a triangle having vertices A\((-5, 6)\), B\((3, -4)\), and C\((3, 3)\)
   □(2) a triangle having vertices A\((5, 4)\), B\((-1, 2)\), and C\((2, -4)\)
   □(3) a quadrilateral having vertices A\((-3, 4)\), B\((-5, -1)\), C\((-2, -4)\), and D\((2, 1)\)

2. Imagine a parallelogram ABCD. When the coordinates of vertices A, B, and C are the following, find the coordinates of vertex D.
   □(1) A\((2, 4)\), B\((-3, 1)\), C\((-5, -3)\)
   □(2) A\((0, 6)\), B\((3, -3)\), C\((-2, -5)\)

3. There are two graphs in the figure on the right. One is a graph of the function \(y = 2x\) with a domain of \(x \geq 0\), and the other is a graph of the function \(y = \frac{a}{x}\) with a domain of \(x > 0\). They intersect at point A, whose \(x\)-coordinate is 4. Point B is on the graph of \(y = \frac{a}{x}\) and its \(x\)-coordinate is 16. Answer the following questions.
   □(1) Find the value for \(a\).
   □(2) Find the area of triangle AOB.

4. As shown in the figure on the right, point P moves on the graph of the function \(y = x\) with a domain of \(x > 0\). A perpendicular line dropped from P to the \(x\)-axis intersects with the graph of the function \(y = -x\) at point Q. Supposing you make a square PQRS with PQ as a side, answer the following questions.
   □(1) Letting \(t\) be the \(x\)-coordinate of point P, express the length of PQ in terms of \(t\).
   □(2) When the length of PQ is 8, find the coordinates of point S.
   □(3) When the \(x\)-coordinate of point S is 27, find the coordinates of point P.
Chapter 4  Proportion and inverse proportion

Comprehension test for Chapter 4

1 Proportion and inverse proportion  Write an expression to indicate the following relationship between $x$ and $y$. If $y$ is proportional to $x$, mark a circle (⊙). If $y$ is inversely proportional to $x$, mark a triangle (△). If $y$ is neither proportional nor inversely proportional to $x$, mark a cross (×).

□(1) It took $y$ minutes to fill up a 100-L tank by pouring in water at a rate of $x$ L per minute.

□(2) When a 15-cm candle burns and becomes $x$ cm shorter, it is $y$ cm long.

□(3) A piece of wire weighs 20 g per meter. An $x$-cm piece of this wire weighs $y$ g.

2 Coordinates  Find the coordinates of the following points.

□(1) the reflections of the point $(5, 10)$ about the $x$-axis, about the $y$-axis, and about the origin

□(2) the point plotted by moving the point $(4, -7)$ 9 spaces to the left and 10 spaces up

□(3) the midpoint between the points $(-6, 7)$ and $(4, 3)$

3 Expressions and graphs, Domains  Answer the following questions.

□(1) $y$ is proportional to $x$. When $x=3$, $y=24$. Express $y$ in terms of $x$.

□(2) Write an expression of the graph of a hyperbola that passes through the point $(-2, -11)$.

□(3) Think about the proportion $y=-4x$. When the domain of $x$ is $-1≤x≤1$, find the domain of $y$.

□(4) Think about the inverse proportion $y=-\frac{10}{x}$. When $y=2$, find the value for $x$.

4 Graphs and figures  In the figure on the right, point $P$ is on the graph of the function $y=\frac{3}{4}x$. Imagine two straight lines that pass $P$: one line is parallel to the $y$-axis, and the other line is parallel to the $x$-axis. Label as $Q$ and $R$ the points where the two lines intersect with the graph of the function $y=-\frac{3}{2}x$. Answer the following questions, on condition that the $x$-coordinate of point $P$ is a positive number.

□(1) When the $x$-coordinate of point $P$ is 8, find the area of triangle PQR.

□(2) When the length of PQ is 9, find the $x$-coordinate of point P.
End-of-chapter problems

1. There is a rectangle ABCD. The lengths of AB and AD are 10 cm and 20 cm, respectively. Point P starts from vertex B and move to vertex C on side BC at a speed of 1 cm per second. Letting $y \text{ cm}^2$ be the area of triangle ABP $x$ seconds after point P starts, answer the following questions.

□(1) Express $y$ in terms of $x$. Then fill in the blank of the inequality below that shows the domain of $x$.

\[0 \leq x \leq \square\]

□(2) Graph the relationship between $x$ and $y$.

□(3) When $y=75$, find the value for $x$.

2. There are three points A(2, 4), B(−2, −1), and C(5, −3), as shown in the figure on the right. Answer the following questions.

□(1) Write an expression to indicate the graph of a proportional relationship that passes through the midpoint between B and C.

□(2) When you make a parallelogram ADBC so that AB is one of its diagonals, find the coordinates of vertex D.

□(3) Find the area of triangle ABC, letting a space along the coordinate axes be 1 cm.

3. There are three graphs in the figure on the right. The curve $\ell$ is the graph of $y=\frac{a}{x}$ with a domain of $x>0$. The straight lines $m$ and $n$ are the graphs of $y=2x$ and $y=−x$, respectively. $\ell$ and $m$ intersects at point P, whose $x$-coordinate is 3. Point Q is on line $n$ and its $x$-coordinate is a positive number. Answer the following questions.

□(1) Find the value for $a$.

□(2) When the midpoint between P and Q is on the $x$-axis, find the coordinates of Q.

□(3) When the area of triangle OPQ is 18, find the coordinates of Q.
Chapter 5  Plane figures

Key points of study

1. Lines and angles
   (1) A line that passes through two points A and B is called line AB.
   (2) A portion of line AB that has two endpoints A and B is called segment AB.
   (3) A portion of line AB that starts at A and goes off in the direction of B to infinity is called ray AB.
   (4) The halfway point of a segment is called the midpoint.
   (5) Triangle ABC is written as \( \triangle ABC \), using the symbol \( \triangle \).
   (6) The angle formed by ray OA and ray OB is called angle AOB, which is written as \( \angle AOB \).
   (7) When lines AB and CD are parallel, the relationship is expressed as \( AB \parallel CD \). When they are perpendicular, the relationship is expressed as \( AB \perp CD \).

2. Circle arcs and chords, Tangents
   (1) When points A and B are on the circumference of a circle, the part from A to B is called arc AB, which is written as \( \overline{AB} \).
   The segment that links A and B is called chord AB.
   (2) When line \( \ell \) is tangent to circle O at point A, line \( \ell \) is called a tangent of circle O and point A is called the tangent point. Line \( \ell \) is perpendicular to OA (\( \ell \perp OA \)).

3. Figure transformation
   (1) Figure construction means drawing certain figures only with a straightedge and a compass. A straightedge is used to draw straight lines. A compass is used to draw circles and copy the length of segments.
   (2) When solving construction problems, do not erase the lines or parts of circles you draw to show the process of constructing the figures.

4. Sector arc length and area
   (1) A part of a circle bounded by two radii is called a sector.
   (2) The angle made by the two radii in the sector is called the central angle.
   (3) For a sector with radius \( r \), central angle \( \alpha \), arc length \( \ell \), and area \( S \), you can set up the following formulas.
   \[ \ell = 2\pi r \times \frac{\alpha}{360} \]
   \[ S = \pi r^2 \times \frac{\alpha}{360}, \quad S = \frac{1}{2} \ell r \]
Let's learn the basics 1 Lines and angles

1. Lines, segments, and rays
   - Line AB
   - Segment AB
   - Ray AB
   - Ray BA

2. When writing an expression to indicate the relationships between the lengths of segments,
   - If the length of segment AB is equal to that of segment BC: \( AB = BC \)
   - If the length of segment AC is twice that of segment AB: \( AC = 2AB \)

3. The halfway point of a segment, for example point B in the figure on the right, is called the midpoint.

How to express angles and triangles

1. In the figure on the right, angle ABC is written
   - \( \angle ABC \) (or \( \angle CBA \) or \( \angle B \)), using the symbol \( \angle \).

2. Triangle ABC is written as \( \triangle ABC \), using the symbol \( \triangle \).

1. As shown in the figure on the right, there are points A to F on a plane.
   - Draw the following line by linking two points.
     - (1) Line BD
     - (2) Line DC
     - (3) Segment AB
     - (4) Segment BC
     - (5) Ray BE
     - (6) Ray EF

2. Answer the following questions about the rectangle and triangle on the right.
   - (1) Express angles (1) to (3) in Figure 1 using the symbol \( \angle \).
   - (2) Which angle corresponds with the following (1) to (3) in Figure 2?
     - Answer using numbers (i) to (vi).
     - (i) \( \angle PRQ \)    (ii) \( \angle RPS \)    (iii) \( \angle PSR \)
   - (3) State all triangles in Figure 2 using the symbol \( \triangle \).
   - (4) Write an expression to indicate that the length of segment AD is equal to that of segment BC in Figure 1.
   - (5) In Figure 1, how many centimeters is the distance between point A and point C?
   - (6) In Figure 2, when QR = RS, point R is what of segment QS?
Let's learn the basics

(1) When two lines on the same plane do not intersect, they are said to be parallel. When lines $AB$ and $CD$ are parallel, the relationship is expressed as $AB \parallel CD$, using the symbol $\parallel$.

(2) When two lines make a right angle at the point they intersect, they are said to be perpendicular. When lines $AB$ and $CD$ are perpendicular, the relationship is expressed as $AB \perp CD$, using the symbol $\perp$.

3 Lines that pass through points $A$, $B$, $C$, $D$, and $E$ are given in the figure on the right. Answer the following questions.

(1) Express the positional relationship between line $BD$ and line $CE$ using a symbol.

(2) Express the positional relationship between line $AC$ and line $AE$ using a symbol.

(3) Suppose you draw line $DF$, which passes through point $D$ and is perpendicular to line $AE$. Express the positional relationship between line $DF$ and line $AC$ using a symbol.

(4) Suppose you draw line $AG$, which passes through point $A$ and is perpendicular to line $BD$. Express the positional relationship between line $AG$ and line $CE$ using a symbol.

4 Lines $\ell$, $m$ and points $A$ to $E$ are given in the figure on the right. When the scale of the grid is 1 cm, answer the following questions.

(1) Which point is the closest to line $\ell$?

(2) How many cm is the distance between point $A$ and line $\ell$?

(3) How many cm is the distance between line $\ell$ and line $m$?

Points

1. When two lines are perpendicular, one line is called the line perpendicular to the other.
2. Suppose you draw a line perpendicular to line $\ell$ from point $A$, which is not on line $\ell$, and let the intersection be $H$. The length of segment $AH$ is said to be the distance between point $A$ and line $\ell$.
3. If two lines are parallel, the distance between any point on one line and the other line is always the same. This fixed distance is called the distance between the two parallel lines.
**Let's learn the basics**

### Circle arcs and chords, Tangents

**Circle arcs and tangents**

In the figure of a circle on the right, the part of the circumference from A to B is called **arc AB**, which is written as \( \overline{AB} \). The segment that links A and B is called **chord AB**.

---

**Property of a tangent**

When a line shares only one point with a circle, the line is said to be **tangent** to the circle. This line is called a **tangent** of the circle, and the point shared is called the **tangent point**.

A tangent of a circle is perpendicular to the radius that passes through its tangent point. For example, you can say \( \ell \perp OM \) in the figure on the right.

---

### Answer the following questions about the figure on the right.

1. Use a symbol to express the part shown with the thick line.

2. What kind of shape is created by linking O with A, O with B, and A with B? It becomes a particular shape when the length of chord AB is equal to the radius of circle O. What is it?

3. Draw the longest chord in the figure on the right.

---

### In the figure on the right, A and B are the tangent points where the tangents drawn from point P are tangent to circle O. Answer the following questions.

1. Express the positional relationship between segment OB and line BP using a symbol.

2. Find the size of \( \angle x \).

---

### In the figure on the right, line PA and line PB are the tangents of circle O. They are tangent to circle O at point A and point B respectively. Line PA and line BO intersect at point C. When OA=AC, find the size of the following angles.

1. \( \angle OAC \)
2. \( \angle AOC \)
3. \( \angle AOB \)
4. \( \angle APB \)
Let's learn the basics Properties of symmetric figures

Figures that have line symmetry
In a figure that has line symmetry, the axis of reflection perpendicularly intersects the segments linking corresponding points and divide them into equal halves.

For example, in the figure on the right, the positional relationship between segment DE and line \( \ell \) is expressed as \( DE \perp \ell \) with a symbol \( \perp \). The relationship between the length of segment AI and that of segment HI is expressed as \( AI = HI \).

Figures that have point symmetry
In a figure that has point symmetry, the center of symmetry is the intersection of two segments linking corresponding points. It is also the midpoint of both segments.

For example, in the figure on the right, the positional relationship between the length of segment AO and that of segment GO is expressed as \( AO = GO \).

8 The figure on the right has line symmetry and line \( \ell \) is the axis of reflection. Answer the following questions.

- (1) Which side corresponds to side AB?

- (2) Express the positional relationship between segment BC and line \( \ell \) using a symbol.

- (3) Write an expression to indicate the relationship between the length of segment BE and that of CE.

- (4) When BC = 6 cm, find the length of segment BE.

9 The figure on the right has point symmetry and point O is the center of symmetry. Answer the following questions.

- (1) Which point corresponds to point C?

- (2) Write an expression to indicate the relationship between the length of segment OA and that of OE.

- (3) Write an expression to indicate the relationship between the length of segment HD and that of OD.

- (4) When OA = 4 cm, find the length of segment AE.
1. In the parallelogram ABCD on the right, segment AE drawn from vertex A intersects side BC perpendicularly. Answer the following questions.

1) Express the positional relationship between side AB and side DC using a symbol.

2) Express the positional relationship between segment AE and side BC using a symbol.

3) Imagine two points that divide side BC into three equal parts. When point E is one of them and the closer to B, write an expression to indicate the relationship between the length of segment BC and that of BE.

2. Line \( \ell \), line \( m \) and points A to F are given in the figure on the right. When the scale of the grid is 1 cm, answer the following questions.

1) How many cm is the distance between point F and line \( \ell \)?

2) Which two points among A to F are at the same distance from line \( m \)?

3) How many cm is the distance between line \( \ell \) and line \( m \)?

3. In the figure on the right, A and B are the tangent points where the tangents drawn from point P are tangent to circle O. Answer the following questions.

1) Use a symbol to express the part shown with the thick line.

2) When \( \angle AOB = 130^\circ \), find the size of \( \angle x \).

3) When \( \angle x = 90^\circ \), what kind of figure is quadrangle AOBP?

4. In the following figures, points A and B are the tangent points where the lines are tangent to circle O. Find the size of \( \angle x \).

1) \[ \angle x = 26^\circ \]

2) \[ \angle x = 38^\circ \]

3) \[ \angle x = 38^\circ \]
Let's learn the basics  

1 Translation

Moving a figure by sliding it in a single direction for a certain distance is called a translation.

① All segments linking pairs of corresponding points are parallel and have the same length. For example, in the figure on the right, \( \overline{AD} \parallel \overline{CF} \parallel \overline{BE} \) and \( \overline{AD} = \overline{CF} = \overline{BE} \).

② All corresponding sides are parallel. For example, in the figure on the right, \( \overline{AB} \parallel \overline{DE} \), \( \overline{BC} \parallel \overline{EF} \), and \( \overline{CA} \parallel \overline{FD} \).

1 Translate \( \triangle ABC \) so that vertex A comes to the position of D and draw the corresponding \( \triangle DEF \).

2 In the figure on the right, \( \triangle ABC \) and \( \triangle PQR \) are congruent. Answer the following questions.

① Fill in the blanks, noting that the scale of the grid is 1 cm.

\[ \triangle PQR \text{ is } \_ \_ \_ \_ \_ \text{ from } \triangle ABC \text{ in the direction of } A \text{ to } P \text{ by } \_ \_ \_ \_ \text{ cm.} \]

② Which point corresponds to point C?

③ Which angle corresponds to \( \angle P \)?

④ Which side corresponds to side AB?

⑤ Write an expression to indicate the relationship between side BC and side QR.

⑥ Write an expression to indicate the relationship between segment AP and segment BQ.
Let's learn the basics 2 Rotation

Moving a figure by turning it at a certain angle around a central point O is called a rotation. The central point O is the center of rotation. Rotating a figure 180° is specially called point-symmetry rotation.

1. All pairs of corresponding points are at the same distance from the center of rotation. For example, in the figure on the right, OA = OD, OB = OE, and OC = OF.

2. The size of the angle made by linking each pair of corresponding points and the center of rotation is equal. For example, in the figure on the right, \( \angle AOD = \angle BOE = \angle COF \). It is also equal to the size of the angle of rotation.

3. Answer the following questions.

□(1) Rotate \( \triangle ABC \) 180° around the center of rotation O and draw the corresponding \( \triangle PQR \).

□(2) Rotate \( \triangle ABC \) 90° clockwise around the center of rotation O and draw the corresponding \( \triangle STU \).

4. In the figure on the right, \( \triangle DEF \) is rotated 40° from \( \triangle ABC \) around the center of rotation O. Answer the following questions.

□(1) Which point corresponds to point A?
□(2) Which side corresponds to side BC?
□(3) Which segment has the same length as segment AO?
□(4) When \( \angle BOA = 25° \), find the size of \( \angle AOE \).

5. In the figure on the right, \( \triangle DBE \) is rotated from \( \triangle ABC \). Answer the following questions.

□(1) Which point is the center of rotation?
□(2) Which side has the same length as side AC?
□(3) What is the size of the angle of rotation?
Let's learn the basics  3  Reflection

Moving a figure by turning it over a single central line \( \ell \) is called a reflection. This central line \( \ell \) is called the axis of reflection.

The axis of reflection is the perpendicular bisector of the segments linking pairs of corresponding points. For example, in the figure on the right, \( AP = DP, BQ = EQ, CR = FR, AD \perp \ell, BE \perp \ell, \) and \( CF \perp \ell. \)

6 In the following figure, draw a triangle by reflecting \( \triangle ABC \) over the axis of reflection \( \ell. \)

\[
\begin{array}{c}
\text{(1)} \\
\text{(2)}
\end{array}
\]

7 In the figure on the right, \( \triangle DEF \) is reflected from \( \triangle ABC \) over the axis of reflection \( \ell. \) Answer the following questions.

\(\Box\) (1) Which side corresponds to side \( AC? \)
\(\Box\) (2) Which angle has the same size as \( \angle B? \)
\(\Box\) (3) Express the positional relationship between segment \( AD \) and line \( \ell \) using a symbol.
\(\Box\) (4) When segment \( BE \) and line \( \ell \) intersect at point \( M, \) write an expression to indicate the relationship between the length of segment \( BM \) and segment \( EM. \)

8 In the figure on the right, \( \triangle DEF \) is reflected from \( \triangle ABC \) over the axis of reflection \( \ell. \) Answer the following questions.

\(\Box\) (1) Which point and side correspond to point \( B \) and side \( EF \) respectively?
\(\Box\) (2) When segment \( AD \) and line \( \ell \) intersect at point \( M, \) find the length of segment \( DM. \)
\(\Box\) (3) When point \( P \) is on line \( \ell, \) write an expression to indicate the relationship between the length of segment \( PA \) and segment \( PD. \)
1. In the figure on the right, draw a figure by transforming quadrilateral ABCD as follows.
   □(1) Translate the quadrilateral so that vertex D comes to the position of D'.
   □(2) Reflect the quadrilateral over the axis of reflection ℓ.

2. In the figure on the right, the coordinates of point P are (-3, 5).
   When you rotate point P, 90° in the direction of the arrow around the origin O, find the coordinates of the new point.

3. In the figure on the right, the inside of rectangle ABCD is divided into 8 congruent triangles. Answer the following questions.
   □(1) When ΔAEI is translated, which triangle does it overlap?
   □(2) When ΔDGI is rotated, which triangle does it overlap?
   □(3) Find all triangles which overlap ΔIHA when it is reflected only once.

4. In the figure on the right, ΔDEF is reflected from ΔABC over the axis of reflection ℓ. ΔGHI is reflected from ΔDEF over the axis of reflection m. When ∠P is 45°, answer the following questions.
   □(1) Find the size of ∠CPL.
   □(2) How can ΔABC be transformed to ΔGHI in one operation?
Let's learn the basics  1 Constructing triangles

Conditions to determine the shape of a triangle... If you know one of the following information on a triangle, you can determine its shape.

(i) Length of three sides
(ii) Length of two sides and the size of the angle between them
(iii) Length of one side and the size of the angles on both ends

Question Construct the following $\triangle ABC$.

1. $AB=6\text{ cm}$, $BC=8\text{ cm}$, $CA=7\text{ cm}$
2. $AB=8\text{ cm}$, $BC=8\text{ cm}$, $\angle B=30^\circ$
3. $BC=8\text{ cm}$, $\angle B=45^\circ$, $\angle C=60^\circ$

Solution You can construct each figure as follows.

1. (1) Draw segment BC with length 8 cm.
   1. Label as A the point where it intersects the circle centered at C with radius 7 cm.

2. (2) Draw segment BC with length 8 cm.
   2. Draw a ray from B so that $\angle B=30^\circ$.
   3. Label as A the point where it intersects the circle centered at B with radius 6 cm.

3. (3) Draw segment BC with length 8 cm.
   2. Draw a ray from C so that $\angle C=60^\circ$ and label as A the point where it intersects B.

Answer

1. Construct the following $\triangle ABC$.

   □ (1) $AB=6\text{ cm}$, $BC=7\text{ cm}$, $CA=5\text{ cm}$
   □ (2) $AB=6\text{ cm}$, $BC=7\text{ cm}$, $\angle B=50^\circ$
   □ (3) $BC=7\text{ cm}$, $\angle B=60^\circ$, $\angle C=30^\circ$
   □ (4) $AB=AC=6\text{ cm}$, $\angle B=45^\circ$

2. Construct $\triangle ABC$ so that $BC=8\text{ cm}$, $\angle B=40^\circ$, and $CA=6\text{ cm}$. Is it possible to determine the shape of this triangle?
**Let's learn the basics**  
**2. Constructing perpendicular bisectors**

A line that is perpendicular to a segment and passes through its midpoint is called the **perpendicular bisector** of the segment.

Any point on the perpendicular bisector of segment $AB$ is at the same distance from points $A$ and $B$.

---

**Constructing the perpendicular bisector of segment $AB$**

1. Draw two circles with equal radii centered at the two segment endpoints $A$ and $B$.
2. Label as $P$ and $Q$ the two points where the circles intersect and draw line $PQ$. This line is the perpendicular bisector of segment $AB$.

---

3. Construct the perpendicular bisector of segment $AB$ below.

4. In the figure on the right, construct three points which divide segment $AB$ into four equal parts. (Label them as $P$, $Q$, and $R$ from left to right.)

5. $\triangle ABC$ is given on the right. Construct midpoints $M$ and $N$ of sides $AB$ and $AC$ respectively, and connect $M$ and $N$ to draw segment $MN$. 
6. Answer the following questions about $\triangle ABC$ on the right.

(1) Construct three perpendicular bisectors of sides $AB$, $BC$, and $CA$.

(2) What does the result of (1) suggest?

---

7. Segments $AB$ and $BC$ are given in the figure on the right. Answer the following questions.

(1) Construct two perpendicular bisectors of segments $AB$ and $BC$.

(2) When you label as $P$ the intersection of the two perpendicular bisectors of (1), what can be said about $P$?

---

8. Line $\ell$ and two points $A$ and $B$ are given in the figure below. Construct point $P$ so that it is on line $\ell$ and at the same distance from points $A$ and $B$.

(1)

(2)

---

9. In the figure on the right, construct point $P$ so that it is on the circumference of circle $O$ and at the same distance from points $A$ and $B$.

---

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Let's learn the basics  Constructing angle bisectors

A ray that divides an angle into two equal parts is called the angle bisector. Any point on the angle bisector of $\angle XOY$ is at the same distance from sides $OX$ and $OY$.

Constructing the angle bisector of $\angle XOY$

1. Draw a circle centered at vertex $O$ of $\angle XOY$, and label as $A$ and $B$ the points where the circle intersects lines $OX$ and $OY$.
2. Draw two circles with equal radii centered at points $A$ and $B$, and label as $P$ the point where they intersect.
3. Draw ray $OP$. This ray is the angle bisector of $\angle XOY$.

10 Construct the bisector of $\angle AOB$ below.

11 In the figure on the right, divide $\angle AOB$ into four equal parts.

12 In the figure on the right, construct $OP$ and $OQ$ so that they are the bisectors of $\angle AOC$ and $\angle BOC$ respectively, and find the size of $\angle POQ$. 
13 Answer the following questions about \( \triangle ABC \) on the right.

\( \square (1) \) Construct the bisectors of \( \angle A, \angle B, \) and \( \angle C \).

\( \square (2) \) What does the result of (1) suggest?

14 \( \triangle ABC \) is given on the right. Construct point \( P \) so that it is on

\( \square \) side \( AC \) and at the same distance from sides \( AB \) and \( BC \).

15 Segment \( AB \) and rays \( AD \) and \( BC \) are given on the right.

\( \square \) Construct point \( O \) so that it is at the same distance from \( AB, BC, \) and \( AD \).

16 In the figure on the right, construct point \( P \) so that it is on

\( \square \) the circumference of circle \( O \) and at the same distance from lines \( AB \) and \( CD \).
Let's learn the basics 4 Constructing perpendicular lines

Draw a line that is perpendicular to line \( \ell \) and passes through point O on line \( \ell \)

1. Draw a circle centered at O and label as A and B the points where it intersects line \( \ell \).
2. Draw two circles centered at A and B with the same radius, and label as P one of the points where they intersect.
3. Draw line OP.
In this way, you can draw line OP that is perpendicular to line \( \ell \).

Draw a line that is perpendicular to line \( \ell \) and passes through point P that is not on line \( \ell \)

1. Draw a circle centered at P and label as A and B the points where it intersects line \( \ell \).
2. Draw two circles centered at A and B with the same radius, and label as Q one of the points where they intersect.
3. Draw line PQ.
In this way, you can draw line PQ that is perpendicular to line \( \ell \).

17 Construct a line that is perpendicular to line \( \ell \) and passes through point P.

\[
\begin{array}{c}
\overline{\ell} \\
\overline{\ell} \\
\overline{\ell}
\end{array}
\]

18 In the figures below, side BC is the base of \( \triangle ABC \). Construct segment AH so that it passes through vertex A and represents the height of this triangle.

19 Answer the following questions about \( \triangle ABC \) on the right.

1. Draw three lines from vertices A, B, and C so that they are perpendicular to sides BC, CA, and AB, respectively.

2. What does the result of (1) suggest?
Let's learn the basics 5 Constructing angles

Question Construct the angles of 90° and 45°, making use of perpendicular lines and angle bisectors.

Solution To construct the angle of 90°, draw a line from point O on line \( \ell \) so that it passes through O and is perpendicular to line \( \ell \), as shown in the figure on the right.

To construct the angle of 45°, since it is half the size of 90°, draw the bisector of 90°.

Answer See the figure on the right.

20 Construct the following angles.

\[ \square \begin{array}{c} 1) \ 90^\circ \\ 2) \ 45^\circ \\ 3) \ 225^\circ \end{array} \]

21 In the figure on the right, construct an isosceles right triangle ABC so that segment AB is one side and \( \angle C = 90^\circ \).

22 Answer the following questions about segment AB on the right.

\( \square \begin{array}{c} 1) \text{ Construct an equilateral triangle so that } AB \text{ is one side and vertex } C \text{ is above it.} \\ 2) \text{ Construct the angle of } 30^\circ, \text{ using the figure you made for (1).} \end{array} \)

23 Answer the following questions about line AB on the right.

\( \square \begin{array}{c} 1) \text{ Construct an equilateral triangle } ABC \text{ so that } AB \text{ is one side and vertex } C \text{ is above it. Then construct } \text{ray } BD \text{ so that it is perpendicular to line } AB \text{ and point } D \text{ is above line } AB. \\ 2) \text{ Construct ray } BE \text{ so that } \angle CBE = 75^\circ, \text{ using the figures of (1).} \end{array} \)
Let's learn the basics 6 Circles and figure construction

**Question**: Construct circle $O$ so that it has points $A$, $B$, and $C$ on its circumference.

**Solution**: $A$, $B$, and $C$ should be at the same distance from center $O$, so $OA = OB = OC$. Since $OA = OB$, $O$ is on the perpendicular bisector of segment $AB$. Likewise, since $OB = OC$, $O$ is on the perpendicular bisector of segment $BC$. Therefore, center $O$ is where the perpendicular bisectors of segments $AB$ and $BC$ intersect. You can draw a circle centered at $O$ with radius $OA$.

**Answer**: See the figure on the right.

24 In the figure on the right, construct circle $O$ so that it has points $A$, $B$, and $C$ on its circumference.

25 In the figure on the right, construct circle $O$ so that center $O$ is on line $\ell$ and points $A$ and $B$ are on its circumference.

26 In the figure on the right, construct the tangent to circle $O$ so that it passes through point $P$.

27 In the figure on the right, construct circle $O$ so that it is tangent to line $\ell$ at point $A$ and has point $B$ on its circumference.

28 In the figure on the right, construct circle $O$ so that its center is on segment $AB$, and it is tangent to two rays $OX$ and $OY$.
Chapter 5  Plane figures

Let's learn the basics  Symmetry and figure construction

**Question**  Point P and line $\ell$ are given in the figure on the right. Construct point Q so that P and Q are symmetric about line $\ell$.

**Solution**  You can construct point Q as follows.

1. Draw a line so that it is perpendicular to line $\ell$ and passes through point P.
2. Draw a circle centered at that point so that it has P on its circumference.
   - Mark the point where the line intersects line $\ell$.
   - Point Q is where the circle intersects the line perpendicular to line $\ell$.

**Answer**  See the figure on the right.

**Another solution**  You can use the following method, too.

1. Mark two points on line $\ell$.
2. Draw two circles centered at those points so that both of them have point P on their circumferences.
3. Those circles intersect at point P and another point, which is point Q.

29  Answer the following questions.

**1) Construct the point so that point P and that point are symmetric about line $\ell$.**

**2) Complete the figure that has line symmetry with line $\ell$ as the axis of symmetry.**

30  A pentagon-shaped sheet of paper is given on the right.

- Construct the figure created by turning the quadrilateral ACDE over diagonal AC.

31  In the figure on the right, construct point P on line $\ell$ so that the length of AP+BP is the shortest.
Let's learn the basics 8 Transformation and figure construction

Question In the figure on the right, segment PQ is reflected from segment AB. Construct the axis of reflection.

Solution The axis of reflection is the perpendicular bisector of a segment linking corresponding points. Therefore, draw the perpendicular bisector of the segment linking A and P (or B and Q).

Answer See the figure on the right.

32 In the figures (1) and (2) below, the shape on the right is reflected from the shape on the left. Construct the axis of reflection.

(1) (2)

33 Answer the following questions.

(1) In Figure 1 below, segment PQ is rotated from segment AB where points P and Q correspond to points A and B, respectively. Construct the center of rotation O.

(2) In Figure 2 below, \(\triangle PQR\) is rotated from \(\triangle ABC\). Construct the center of rotation O.

34 In the figure on the right, point Q is rotated from point P with a certain point O on line \(\ell\) as the center of rotation. Construct that point O.
1. Triangle ABC is given on the right. Construct point P so that it is at the same distance from vertices A and B and also at the same distance from sides AB and AC.

2. Trapezoid ABCD is given on the right. Construct the segment that passes through vertex A and represents the height of this trapezoid.

3. Point C on segment AB is given on the right. Construct isosceles triangle ACD above segment AB so that segment AC is one side, \( \angle ACD = 120^\circ \), and AC = DC.

4. In the figure on the right, construct the circle that is tangent to ray OA and also tangent to ray OB at point P.

5. A sheet of paper in the form of quadrilateral ABCD is given on the right. Construct the figure made by turning \( \triangle ADC \) over diagonal AC.

6. In the figure on the right, segment PQ is reflected from segment AB. Construct the axis of reflection \( \ell \).
Let's learn the basics of sectors.

A part of a circle bounded by two radii is called a sector, as shown in Figure 1 and Figure 2 on the right.

The angle made by the two radii is called the central angle of the sector.

For a sector with radius $r$, central angle $\alpha$, arc length $L$, and area $S$, you can write the following formulas.

\[ L = \frac{\alpha}{360} \times 2\pi \times r \]
\[ S = \frac{\alpha}{360} \times \pi r^2 \]
\[ S = \frac{1}{2} L r \]

**Question** Find the arc length and area of a sector with radius 12 cm and central angle 135°.

**Solution**

The arc length is \( 2\pi \times 12 \times \frac{135}{360} = 9\pi \) cm.

The area is \( \pi \times 12^2 \times \frac{135}{360} = 54\pi \) cm².

[Another solution] According to the formula \( S = \frac{1}{2} L r \), you can find the area using the arc length as follows.

\[ \frac{1}{2} \times 9\pi \times 12 = 54\pi \text{ cm}^2 \]

**Answer** Arc length \( 9\pi \) cm. Area \( 54\pi \) cm².

1. Find the arc length and the area of a sector with the following radius and central angle.

   - (1) \( 60° \) \( 6 \text{ cm} \)
   - (2) \( 120° \) \( 12 \text{ cm} \)
   - (3) \( 240° \) \( 9 \text{ cm} \)
   - (4) \( 10 \text{ cm} \) \( 18° \)
   - (5) \( 90° \) \( 16 \text{ cm} \)
   - (6) \( 270° \) \( 8 \text{ cm} \)
Chapter 5  Plane figures

2  Answer the following questions.
□(1) Consider a sector with central angle $45^\circ$ and arc length $3\pi$ cm.
   ① Write an equation, letting the radius be $x$ cm.
   ② Find the radius.

□(2) Find the radius of a sector with central angle $120^\circ$ and arc length $6\pi$ cm.

3  Answer the following questions.
□(1) Consider a sector with radius $6$ cm and area $16\pi$ cm$^2$.
   ① Write an equation, letting the central angle be $x^\circ$.
   ② Find the central angle.

□(2) Find the central angle of a sector with radius $10$ cm and area $25\pi$ cm$^2$.

4  Answer the following questions.
□(1) Consider a sector with arc length $4\pi$ cm and area $20\pi$ cm$^2$.
   ① Write an equation, letting the radius be $x$ cm.
   ② Find the radius and central angle.

□(2) Find the radius and central angle of a sector with arc length $8\pi$ cm and area $32\pi$ cm$^2$.

5  Two sectors with radius $6$ cm are given on the right. If the central angle of B is twice as great as that of A, how many times are the arc length and the area of B as great as those of A?
Let's learn the basics 2 Combined figures

Question The figure on the right is a combination of sectors and a square. Find the circumference and area of the shaded part.

Solution The circumference of the shaded part is twice the arc length of the sector with radius 8 cm and central angle 90°.

\[
\left(2\pi \times 8 \times \frac{1}{4}\right) \times 2 = 8\pi \text{ (cm)}
\]

The area is twice that of the shaded part of the figure on the right.

\[
\left(\pi \times 8^2 \times \frac{1}{4} - \frac{1}{2} \times 8^2\right) \times 2 = 32\pi - 64 \text{ (cm}^2\text{)}
\]

Answer Circumference \(\approx 8\pi\) cm, Area \(\approx 32\pi - 64\) (cm²)

6 The figures below are combinations of sectors and a square. Find the circumference and area of the shaded parts.

![Diagram](image1)

7 The figures below are combinations of sectors, a circle, and a square. Find the area of the shaded parts.

![Diagram](image2)

8 The figure on the right is a combination of a rectangle and a semicircle. E is the midpoint of CD. Find the area of the shaded part.
**Let's learn the basics  Rolling figures**

**Question** In the figure on the right, $\triangle ABC$ is an equilateral triangle with side 3 cm. When it rolls 360 degrees without sliding along line $\ell$, find the length of the locus of vertex $A$.

**Solution** The triangle first rotates with $C$ at its center until vertex $A$ comes to $A'$, when you can draw the locus $\overline{AA'}$. Next, it rotates with $A'$ at its center until $B'$ comes to $B''$, when $A''$ does not move. Then it rotates with $B''$ at its center until $C'$ comes to $C'''$, when you can draw the locus $\overline{AA''}$. Since $\overline{AA} = \overline{AA'}$, the total length of the locus of vertex $A$ is $2 \times (2\pi \times 3 \times \frac{120}{360}) = 4\pi$ (cm)

**Answer** $4\pi$ cm

9 In the figure on the right, draw the locus of vertex $A$ when rectangle $ABCD$ rolls once without sliding along line $\ell$.

10 In the figure on the right, $\triangle ABC$ is a right triangle with $AB = 6$ cm, $AC = 3$ cm, and $\angle B = 30^\circ$. When it rolls once without sliding along line $\ell$, answer the following questions.

□ (1) Draw the locus of vertex $A$.

□ (2) Find the length of the locus of vertex $A$.

11 A square with its side 10 cm is given on the right. Circle $O$ with radius 1 cm moves around this square once along its outside. Circle $O'$ with radius 2 cm moves around it once along its inside. Answer the following questions.

□ (1) Draw each of the loci of points $O$ and $O'$.

□ (2) Find the travel distance of $O$ and that of $O'$.

□ (3) Find the area of the part between the sides of the square and the locus of point $O$.  

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1. Find the arc length and area of the sectors below.

- (1)  

- (2)  

- (3)  

2. The figures below are combinations of sectors, circles, and squares. Find the area of the shaded parts.

- (1)  

- (2)  

- (3)  

3. An equilateral triangle with side 8 cm and circle O with radius 2 cm are given on the right. The circle moves around the triangle once along its outside. Answer the following questions.

- (1) Draw the locus of point O, the center of the circle.

- (2) Find the length of that locus.

- (3) Find the area of the part between the sides of the triangle and the locus of point O.

4. The figure on the right shows a square-shaped building viewed from the top. Line BC represents a fence. There is a 7-m rope tied to point C and a dog is tied up to the other end of the rope. Find the area of the space where the dog can move around.
1 Parallel and perpendicular lines  Trapezoid ABCD is given on the right. Points E, F, G, and H are the tangent points where each side is tangent to circle O. Answer the following questions.

- (1) Write an expression to indicate that segments OG and OH have the same length.
- (2) Side AD is parallel to side BC. Express this relationship using a symbol.
- (3) Segment OG is perpendicular to side DC. Express this relationship using a symbol.

2 Figure construction  Construct the following figures.

- (1) Perpendicular bisector of segment AB (Figure 1)
- (2) Angle bisector of ∠AOB (Figure 2)
- (3) Line that passes through point P and is perpendicular to line ℓ (Figure 3)

3 Figure transformation  In the figure on the right, ΔDEF is transformed by reflecting ΔABC over the axis of line ℓ and then rotating 30° clockwise around the center of rotation F. Answer the following questions about ΔDEF and ΔABC.

- (1) Which point corresponds to point A?
- (2) Which side corresponds to side AB?
- (3) Which angle corresponds to ∠DEF?
- (4) When ∠ACB=58°, find the size of ∠AFE.

4 Sectors  Find the arc length and area of the sectors below.

- (1) 6 cm
- (2) 9 cm
1. Three lines $\ell$, $m$, and $n$ are given on the right. Construct the point that is on line $n$ and at the same distance from line $\ell$ and line $m$.

2. In the figure on the right, construct the circle that is tangent to side BC of $\triangle ABC$ and rays BP and CQ.

3. In the figure on the right, point A is given inside $\angle XOY$. You can make $\triangle ABC$ with vertex B on OX and vertex C on OY. To make the circumference of this triangle shortest, where should B and C be? Construct the figure to show their positions.

4. The figures below are combinations of circles and squares. Find the area of the shaded parts.

5. In the figure on the right, $\triangle ABC$ is a right triangle with $\angle B=90^\circ$. $\triangle A'B'C$ is rotated $90^\circ$ from $\triangle ABC$ around the center of rotation C. When $AB=8$ cm, $BC=6$ cm, and $CA=10$ cm, answer the following questions.

   1) Draw the loci of vertex A and vertex B.

   2) Find the area of the figure that is bounded by sides $AB$ and $AB'$ and the loci of A and B.
1. **Sets of points**

Lines or curves can be regarded as sets of points.

1. A set of points on a plane that are at the same distance from point O

   They form (the circumference of) a circle with point O as its center, as shown on the right.

2. Two sets of points on a plane that are at the same distance from line \( \ell \)

   They form two lines parallel to line \( \ell \), as shown on the right.

3. A set of points that are at the same distance from two points

   They form the perpendicular bisector of the segment linking the two points, as shown on the right.

4. A set of points that are at the same distance from the two sides of an angle

   They form the bisector of the angle, as shown on the right.

---

1. **Answer the following questions.**

   □ 1) What kind of line do a set of points form on a plane if all of them are at a distance of 5 cm from point O?

   □ 2) What kind of line do a set of points form on a plane if all of them are 3 cm away from line \( \ell \)?

   □ 3) There is a line \( m \) on a plane, and on line \( m \) is point P. How many points are there that are 4 cm away from point P and 3 cm away from line \( m \)?

---

2. **Answer the following questions.**

   □ 1) There is an isosceles triangle ABC with AB = AC. If you change the length of AB and AC without changing the length of BC as shown in Figure 1, what kind of line is the locus of point A?

   □ 2) If you draw circle Os so that they are tangent to two sides of an angle as shown in Figure 2, what kind of line is the locus of center O?

---

*Figure 1*

*Figure 2*
Circumcircles and incircles of triangles

(1) Circumcircles of triangles
   ① The center of a circumcircle is called its circumcenter.
   ② The circumcenter of a triangle is the intersection of the perpendicular bisectors of its three sides.
   ③ The radius of the circumcircle is the distance between the circumcenter and each vertex.

(2) Incircles of triangles
   ① The center of an incircle is called its incenter.
   ② The incenter of a triangle is the intersection of its three angle bisectors.
   ③ The radius of the incircle is the distance between the incenter and each side.

3 Construct the circumcircles of the triangles below.

4 Construct the incircle of the triangle on the right.

5 In the \( \triangle ABC \) on the right, \( \angle A = 90^\circ \), \( AB = 15 \text{ cm} \), \( BC = 17 \text{ cm} \), and \( CA = 8 \text{ cm} \).

   Find the radius of the incircle of \( \triangle ABC \).
Key points of study

1. Positional relationships between lines and planes
   (1) Positional relationships between two lines in space can be summarized as follows:
   - When they lie on the same plane, either they intersect, or they do not intersect if they are parallel.
   - When they do not lie on the same plane, they are in a skewed position.

   (2) There are three positional relationships between a line and a plane:
   - The line is on the plane.
   - They intersect.
   - They are parallel.

   (3) There are two positional relationships between two planes:
   - They intersect.
   - They are parallel.

2. Prisms, cylinders, pyramids, and cones
   (1) Solids like the ones on the right are prisms. If the bottom face of a prism is a triangle or a quadrilateral etc., it is a triangular prism or a quadrangular prism, etc. respectively. If its bottom face is an equilateral triangle or a square etc., it is a regular triangular prism or a square prism, etc. respectively.

   (2) Solids like the ones on the right are cylinders.

   (3) Solids like the ones on the right are pyramids. If the bottom face of a pyramid is a triangle or a quadrilateral etc., it is a triangular pyramid or a quadrangular pyramid, etc. respectively. If its bottom face is an equilateral triangle or a square etc. and all side faces are congruent isosceles triangles, it is a regular triangular pyramid or a square pyramid, etc. respectively.

   (4) Solids like the one on the right are cones.

3. Volume and surface area of solids
   (1) Volume and surface area of prisms and cylinders
   \[
   \text{Volume} = (\text{Base area}) \times (\text{Height}), \quad \text{Surface area} = (\text{Lateral area}) + (\text{Base area}) \times 2
   \]

   (2) Volume and surface area of pyramids and cones
   \[
   \text{Volume} = \frac{1}{3} \times (\text{Base area}) \times (\text{Height}), \quad \text{Surface area} = (\text{Lateral area}) + (\text{Base area})
   \]

   (3) Volume and surface area of spheres

   For a sphere with radius \( r \), volume \( V \), and surface area \( S \):
   \[
   V = \frac{4}{3} \pi r^3, \quad S = 4\pi r^2
   \]

4. Solids of revolution
   A solid created by revolving a plane figure around a line as the axis is called a solid of revolution. You can think that the side face of cylinder or cone shown on the right is created by revolving segment \( AB \) around the axis. This segment is called the generatrix.

5. Projections of solids
   The shape of a solid can be expressed by a combination of the view directly from the front (front view) and the view directly from the top (top view). This type of figure is called a projection.
**Positional relationships between lines and planes**

**Let's learn the basics**

Given one of the following, you can determine one and only one plane.

(i) Three points that are not on one straight line
(ii) A line and a point that is not on the line
(iii) Two lines that intersect with each other
(iv) Two lines that are parallel to each other

---

**Question**

Mark a circle (O) if each of the following enables you to determine one and only one plane, and mark a cross (X) if it doesn't.

1. Two points A and B in space
2. A line \( l \) and a point A that is on it
3. Two lines \( l \) and \( m \) that intersect with each other
4. Two lines \( l \) and \( m \) that are parallel to each other

**Solution**

1. There is more than one plane that contains two points A and B, as shown in Figure 1.
2. You can determine one and only one plane, as shown in Figure 2.
3. You can determine one and only one plane, as shown in Figure 3.
4. You can determine one and only one plane, as shown in Figure 4.
5. You can determine one and only one plane, as shown in Figure 5.

---

**Answer**

1. ×
2. O
3. ×
4. O
5. O
6. O

---

1. Answer the following questions about the pyramid on the right.
   - 1) How many planes are there that contain two points A and C?
   - 2) How many planes are there that contain three points A, C, and D?
   - 3) How many planes are there that contain four points A, C, D, and E?

2. Mark a circle (O) if each of the following enables you to determine one and only one plane, and mark a cross (X) if it doesn't.
   1. Three points on line \( l \)
   2. Two points on line \( l \) and one point that is not on it
   3. One point on plane \( P \) and one point that is not on it
   4. Two lines \( l \) and \( m \) that are perpendicular to each other
   5. Two points that are on line \( l \) and another line \( m \) that is parallel to line \( l \)
Let's learn the basics

Positional relationships between two lines in space can be summarized as follows:

- When two lines lie on the same plane, either they intersect, or they do not intersect if they are parallel.
- When they do not lie on the same plane, they are in a skewed position.

**Question** Answer the following questions about the cuboid ABCD-EFGH on the right.

1. Find all edges that are parallel to edge AB.
2. Find all edges that are perpendicular to edge AB.
3. Find all edges that are in skewed positions in relation to edge AB.

**Solution**

1. If two lines on one plane don't intersect, they are parallel to each other.
2. When two lines intersect at a right angle, they are perpendicular to each other.
3. If two lines lie on different planes in space like a grade separation, they are in a skewed position.

**Answer**

1. Edges DC, EF, HG  
2. Edges AD, AE, BC, BF  
3. Edges DH, CG, EH, FG

3. Express the positional relationships between the following pairs of edges in the cuboid on the right.

- (1) Edge AD and edge FG  
- (2) Edge DC and edge FG  
- (3) Edge BF and edge EH  
- (4) Edge AE and edge CG  
- (5) Edge AB and edge DC  
- (6) Edge BC and edge DH

4. Answer the following questions about the prism on the right, when its bottom face is trapezoid ABCD with AD//BC and AB⊥BC.

- (1) Find all edges that are parallel to edge EH.
- (2) Find all edges that are in skewed positions in relation to edge DC.
- (3) Find all edges that intersect perpendicularly to edge BC.

5. Mark a circle (○) if each of the following statements about the positional relationships between lines in space is always right, and mark a cross (×) if it isn't.

- (1) If two lines \( \ell \) and \( m \) don't intersect, \( \ell \parallel m \).
- (2) There are three lines \( \ell, m, \) and \( n \). If \( \ell \parallel m \) and \( m \parallel n \), then \( \ell \parallel n \).
- (3) There are three lines \( \ell, m, \) and \( n \). If \( \ell \) and \( m \) intersect perpendicularly and \( \ell \) and \( n \) also intersect perpendicularly, then \( m \parallel n \).
Mark a circle (○) if each of the following statements about the positional relationships between lines is true, and a cross (×) if it isn't.

1. Find all edges that are parallel to face EFGH.
2. Find all edges that are perpendicular to face BFGC.
3. Find all faces that are parallel to edge AF.
4. Find all faces that are perpendicular to edge AF.
5. Find all edges that lie on face ABDP.

Answer the following questions about the cubic ABCD-EFGH on the right.

1. Two lines in a plane that are perpendicular to each other.
2. Two lines in a plane that are parallel and pass through the point where the plane P intersects the plane P.
3. Two lines in a plane that are perpendicular to each other, one of which is contained in a plane, the line is said to be on the plane.

Find all edges that are parallel to edge AB.

Find all edges that are perpendicular to edge AB.

Find all edges that lie on face ABDP.

Find all faces that are parallel to face ABDP.

Find all edges that are perpendicular to face ABDP.

Support the intersections plane P at point A. If line f is perpendicular to plane P, then line f passes through point O, line f is perpendicular to plane P. If line f is parallel to line f, then line f and line f are parallel to each other. Two lines m and n that lie on plane P pass through point O, line f is perpendicular to plane P at point A.
Chapter 6  Space figures

Let's learn the basics  4  Positional relationships between two planes

1. If two planes are both perpendicular to one line, they are parallel to each other.
2. In the figure on the right, two planes $P$ and $Q$ intersect and line $XY$ lies on both planes. When you draw lines $OA$ and $OB$ on planes $P$ and $Q$ respectively so that both lines are perpendicular to line $XY$, $\angle AOB$ is said to be the angle that planes $P$ and $Q$ make. When $\angle AOB$ is $90^\circ$, $P \perp Q$.

---

**Question**  Answer the following questions about the cuboid $ABCD$-EFGH on the right.

1. Find the face that is parallel to face $ABCD$.
2. Find all faces that are perpendicular to face $ABCD$.

**Solution**  (1) If two planes are both perpendicular to one line, they are parallel to each other.

Face $ABCD$ is perpendicular to edge $AE$, and face $EFGH$ is also perpendicular to edge $AE$.

(2) If a plane contains a line that is perpendicular to another plane, the two planes are perpendicular.

Edge $AE$ is perpendicular to face $ABCD$, and edge $AE$ is contained in faces $AEFB$ and $AEHD$. Edge $CG$ is perpendicular to face $ABCD$, and edge $CG$ is contained in faces $BFGC$ and $DHGC$.

**Answer**  (1) Face $EFGH$  (2) Faces $AEFB$, $AEHD$, $BFGC$, $DHGC$

---

8 A prism whose bottom face is an equilateral triangle is given on the right. Answer the following questions.

□[1] Find the face that is parallel to face $ABC$.

□[2] Find all faces that are perpendicular to face $ADEB$.

□[3] Find the size of the angle that faces $BEFC$ and $CFDA$ make.

---

9 Mark a circle (○) if each of the following statements about the positional relationships between planes in space is always right, and mark a cross (×) if it isn’t, noting that $P$, $Q$, and $R$ are different planes.

1. If $P \parallel Q$ and $P \parallel R$, then $Q \parallel R$.
2. If $P \perp Q$ and $P \perp R$, then $Q \parallel R$.
3. If $P \parallel Q$ and $P \perp R$, then $Q \perp R$.
4. If $P \perp Q$ and $Q \perp R$, then $P \perp R$.

---

10 Fill in the blanks.

□[1] When two planes are parallel, any line on one plane is ______ to the other plane.

□[2] When two planes are parallel, any line perpendicular to one plane is ______ to the other plane.

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1. Answer the following questions about the cube on the right.
   □(1) Express the positional relationship between edge AE and edge BF.
   □(2) Express the positional relationship between face DHGC and edge BC.
   □(3) Express the positional relationship between edge AD and edge CG.

2. Answer the following questions about the triangular prism on the right.
   □(1) Find a pair of faces that are parallel to each other.
   □(2) How many faces are there that are perpendicular to face ABC?
   □(3) Find the face that intersects face BEFC but not perpendicularly.

3. A prism whose bottom face is a regular hexagon is given on the right. Answer the following questions.
   □(1) How many edges are perpendicular to face ABCDEF?
   □(2) How many edges are parallel to face EKJD?
   □(3) How many edges are in skewed positions in relation to edge GL?
   □(4) Do edge AF and edge IJ lie on the same plane?

4. Mark a circle (○) if each of the following statements about the positional relationships between lines and planes in space is right, and mark a cross (×) if it isn't.
   ① If two planes are both parallel to one line, they are parallel to each other.
   ② Two lines ℓ and m that do not lie on plane P intersect perpendicularly. If ℓ ⊥ P, then m // P.
   ③ There are three lines ℓ, m, and n. Lines ℓ and m intersect perpendicularly. If lines ℓ and n are in a skewed position, then lines m and n also are in a skewed position.
Let's learn the basics 1  Prisms, cylinders, pyramids, and cones

1  Answer the following questions about the solids 1 and 2 on the right.

☐ (1) Name each solid.

☐ (2) What is the number of each differently-shaped face that makes up each solid?

2  Complete the tables below, noting that the number of vertices of a pyramid includes the number of vertices on the bottom face.

☐ (1) ☐ (2)

<table>
<thead>
<tr>
<th></th>
<th>Number of bottom faces</th>
<th>Number of side faces</th>
<th>Number of edges</th>
<th>Number of vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular prism</td>
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</tr>
<tr>
<td>Quadrangular prism</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Pentagonal prism</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Hexagonal prism</td>
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<tr>
<td>n-polygonal prism</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Number of bottom faces</th>
<th>Number of side faces</th>
<th>Number of edges</th>
<th>Number of vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular pyramid</td>
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<td>Quadrangular pyramid</td>
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<td>Hexagonal pyramid</td>
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<td>n-polygonal pyramid</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

3  Answer the following questions.

☐ (1) A regular hexagon is given in Figure 1. You can create a solid by moving this hexagon a certain distance perpendicularly and then connecting the loci of its vertices. What is the name of the solid?

☐ (2) A circle and two points O and P are given in Figure 2. Point O is right above the center of the circle. Point P moves on the circumference of the circle. When P moves around the circle once, the locus of segment OP forms a face. Name the solid that is bounded by this face and the circle.

Point

1. The side face of a cylinder or a cone is created by revolving a segment around its axis, for example segment OP in 3(2). Such a segment is called the generatrix of a cylinder or cone.
Let's learn the basics 2  Nets of prisms and cylinders

Net of a prism  The figure below is the net of a prism, in which its side faces are arranged laterally. Quadrilateral ABCD is a rectangle. Its width is equal to the circumference of the bottom face of this prism.

Net of a cylinder  The figure below is the net of a cylinder. It is made up of a rectangle and two circles. The rectangle is the side face of this cylinder. Its width is equal to the circumference of the bottom face of the cylinder.

4 Name the solids represented by the following nets.

5 Draw a net of a quadrangular prism whose bottom face is a square with side 4 cm and whose height is 3 cm, arranging the side faces laterally. The scale of the grid is 1 cm.

6 Suppose you draw a net of a cylinder whose bottom face is a circle with radius 2 cm and whose height is 5 cm. Answer the following questions.

1) How many centimeters is the width of the rectangle that represents the side face of this cylinder?
   Let \( \pi \) be 3.14.

2) Draw the net of this cylinder. The scale of the grid is 1 cm.
Let’s learn the basics: Volume and surface area of prisms and cylinders

1. The sum of the area of all faces of a solid is called the **surface area**. The total area of all side faces is called the **lateral area**, and the area of one of the bottom faces is called the **base area**.

2. (Volume of prisms and cylinders) = (Bottom area) \( \times \) (Height)
   (Surface area of prisms and cylinders) = (Lateral area) \( + \) (Base area) \( \times 2 \)

---

**Question 1** Find the volume and surface area of the prism on the right.

**Solution**
Volume: \( \frac{1}{2} \times 3 \times 4 \times 3 = 18 \text{ cm}^3 \)
According to the net, the side faces form a rectangle, and its width is 3 + 4 + 5 = 12 (cm)
Lateral area: 3 \( \times \) 12 = 36 (cm²), Base area = 6 cm². So,
Surface area: (Lateral area) \( + \) (Base area) \( \times 2 \) = 36 + 6 \( \times 2 \) = 48 (cm²)
**Answer** Volume: 18 cm³, Surface area: 48 cm²

**Question 2** Find the volume and surface area of the cylinder on the right.

**Solution**
Volume: \( \pi \times 5^2 \times 8 = 200 \pi \text{ cm}^3 \)
According to the net, the side face forms a rectangle, and its width is 2\( \pi \times 5 \) = 10\( \pi \) (cm)
Lateral area: 8 \( \times \) 10\( \pi \) = 80\( \pi \) (cm²), Base area: 25\( \pi \) cm². So,
Surface area: (Lateral area) \( + \) (Base area) \( \times 2 \) = 80\( \pi \) + 25\( \pi \) \( \times 2 \) = 130\( \pi \) (cm²)
**Answer** Volume: 200\( \pi \) cm³, Surface area: 130\( \pi \) cm²

7 The figures on the right show a triangular prism and its net. Answer the following questions.

1. Find the values for \( x \) and \( y \).
2. Find the volume and surface area of this triangular prism.

---

8 Find the volume and surface area of the following triangular prisms.

1.  

2.  

3.  

---

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9 Find the volume and surface area of the following cuboids.

\( \square 1 \): 6 cm, 6 cm, 4 cm

\( \square 2 \): 5 cm, 7 cm, 12 cm

\( \square 3 \):

\[
\begin{array}{c}
8 cm \\
15 cm \\
10 cm
\end{array}
\]

10 The figures on the right show a cylinder and its net. Answer the following questions.

\( \square 1 \) Find the values for \( x \) and \( y \).

\( \square 2 \) Find the volume and surface area of this cylinder.

11 Find the volume and surface area of the following cylinders.

\( \square 1 \):

\[
\begin{array}{c}
5 cm \\
15 cm
\end{array}
\]

\( \square 2 \):

\[
\begin{array}{c}
4 cm \\
8 cm
\end{array}
\]

\( \square 3 \):

\[
\begin{array}{c}
20 cm \\
12 cm
\end{array}
\]

12 Find the volume and surface area of the following quadrangular prisms.

\( \square 1 \):

\[
\begin{array}{c}
4 cm \\
2 cm \\
5 cm \\
3 cm \\
5 cm \\
3 cm
\end{array}
\]

\( \square 2 \):

\[
\begin{array}{c}
5 cm \\
4 cm \\
5 cm \\
5 cm \\
6 cm \\
11 cm
\end{array}
\]

\( \square 3 \):

\[
\begin{array}{c}
2.4 cm \\
5 cm \\
6.5 cm \\
6 cm \\
3 cm \\
6.5 cm
\end{array}
\]

13 Find the volume and surface area of the following solids.

\( \square 1 \):

\[
\begin{array}{c}
3 cm \\
1 cm
\end{array}
\]

\( \square 2 \):

\[
\begin{array}{c}
8 cm \\
20 cm
\end{array}
\]
Let's learn the basics  Nets of pyramids and cones

Net of a pyramid  The figure below is the net of a regular pentagonal pyramid, in which its side faces are arranged laterally. All side faces are congruent isosceles triangles.

[Diagrams of a net of a pentagonal pyramid and a net of a cone]

14 Name the solids represented by the following nets.

□ 1)  □ 2)  □ 3)

15 Draw a net of the square pyramid below.
□ The scale of the grid is 1 cm.

16 Suppose you draw the net of a cone whose bottom face is a circle with radius 2 cm and whose generatrix is 8 cm long. Answer the following questions.
□ 1)  How many centimeters is the arc length of the sector which makes up the side face of this cone?
□ 2)  Find the size of the central angle of the sector.
□ 3)  Draw the net of this cone. The scale of the grid is 1 cm.
Let's learn the basics 5 Volume and surface area of pyramids

\[(\text{Volume of pyramids}) = \frac{1}{3} \times \text{(Base area)} \times \text{(Height)}, \quad (\text{Surface area of pyramids}) = (\text{Lateral area}) + (\text{Base area})\]

**Question** Find the volume and surface area of the square pyramid on the right.

**Solution**
Volume: \(\frac{1}{3} \times 10^2 \times 12 = 400 \text{ cm}^3\)
Surface area: The base area is 100 cm\(^2\). The side faces are all congruent isosceles triangles, so
\(\left(\frac{1}{2} \times 10 \times 13\right) \times 4 + 100 = 360 \text{ cm}^2\)

**Answer** Volume: 400 cm\(^3\). Surface area: 360 cm\(^2\)

17 Find the volume of the following pyramids.

18 Find the surface area of the following pyramids.

19 Find the volume of the following solids.

20 A cube whose side is 12 cm is given on the right. When you cut this cube with a plane that passes through three vertices A, C, and F and take off the smaller solid, find the volume of the remaining solid.
**Chapter 6  Space figures**

---

Let's learn the basics 6 Volume and surface area of cones

### Question 1
Find the volume of the cone on the right.

**Solution**

\[
\text{Volume of cones} = \frac{1}{3} \times \text{(Base area)} \times \text{(Height)}.
\]

\[
\frac{1}{3} \times (\pi \times 3^2) \times 6 = 18\pi \text{ cm}^3
\]

**Answer**

18\pi \text{ cm}^3

---

1. (Surface area of pyramids and cones) = (Lateral area) + (Base area)
2. Letting \( \ell \) be the circumference of the bottom face of the cone (the arc length of the sector that makes up the side face), \( R \) be the generatrix length, and \( S \) be the lateral area, you can write the formula \( S = \frac{1}{2} \ell R \).
3. Letting \( r \) be the radius of the bottom face of the cone, \( \ell \) gives you \( S = \frac{1}{2} \times 2\pi r \times R \), so \( S = \pi r R \).
4. The central angle of the sector that makes up the side face is \( \theta = \frac{r}{R} \times 360^\circ \).

---

### Question 2
Find the surface area of the cone on the right.

**Solution**

Letting \( a \) be the central angle of the sector which makes up the side face,

\[
2\pi \times 8 \times \frac{a}{360} = 2\pi \times 4, \ a = 180.
\]

You can draw the net on the lower right.

Lateral area \( \cdots \times 8 \times \frac{180}{360} = 32\pi \text{ cm}^2 \), Base area \( \cdots \times 4^2 = 16\pi \text{ cm}^2 \)

Surface area \( \cdots \text{(Lateral area)} + \text{(Base area)} = 32\pi + 16\pi = 48\pi \text{ cm}^2 \)

**[Note]** If you use the formula \( \text{3} \) above, you can find the lateral area by calculating \( \pi \times 4 \times 8 = 32\pi \text{ cm}^2 \). Also, if you use the formula \( \text{4} \) above, you can find the central angle of the sector by calculating \( \frac{4}{8} \times 360^\circ = 180^\circ \).

**Answer**

48\pi \text{ cm}^2

---

21 Find the volume of the following cones.

\[ \square 1 \]

\[ \square 2 \]

\[ \square 3 \]

22 Answer the following questions about the cone on the right.

\[ \square 1 \] Find the base area.

\[ \square 2 \] Find the central angle of the sector that makes up the side face.

\[ \square 3 \] Find the lateral area.

\[ \square 4 \] Find the surface area.
23 Find the surface area of the following cones.

(1) 

24 Find the surface area of the cones whose nets are as follows.

(1) 

25 Find the volume of the following solids.

(1) 

26 Find the volume and surface area of the following solids.

(1) 

2 Prisms, cylinders, pyramids, cones, and spheres
Chapter 6  Space figures

Let's learn the basics ▶ Volume and surface area of spheres

For a sphere with radius \( r \), volume \( V \), and surface area \( S \),

\[
V = \frac{4}{3} \pi r^3 \quad S = 4\pi r^2
\]

---

**Question**  Find the volume and surface area of a sphere whose radius is 3 cm.

**Solution**  The volume is \( \frac{4}{3} \pi \times 3^3 = 36\pi \) (cm\(^3\)).

The surface area is \( 4\pi \times 3^2 = 36\pi \) (cm\(^2\)).

**Answer**  Volume \( = 36\pi \) cm\(^3\), Surface area \( = 36\pi \) cm\(^2\)

27  Find the volume and surface area of the following spheres.

- (1) Sphere whose radius is 2 cm
- (2) Sphere whose radius is 9 cm
- (3) Sphere whose diameter is 8 cm
- (4) Sphere whose diameter is 20 cm

28  Find the volume and surface area of the following solids.

- (1) Hemisphere
- (2) Quarter of a sphere
- (3) Combination of a hemisphere and a cylinder

29  A sphere, a cylinder, and a cone are given on the right. The radius of the sphere is 3 cm. The cylinder is just the right size to put the sphere in. The cone is just the right size to fit in the cylinder. Answer the questions below.

- (1) Express the volume ratio between these three solids in the form of “Sphere: Cylinder : Cone.”

- (2) Express the surface area ratio of the sphere to the cylinder in the form of “Sphere : Cylinder.”
Let's learn the basics 8 Solids of revolution

1. A solid created by revolving a plane figure around a line as the axis is called a solid of revolution.
2. When you regard a cylinder or a cone as a solid of revolution created by revolving a rectangle or a right triangle, the segment that revolves to make up the side face is called the generatrix.

Question Find the volume and surface area of the solid that is created when the shape below is revolved once around the axis of revolution line $\ell$.

<table>
<thead>
<tr>
<th>(1) Rectangle</th>
<th>(2) Right triangle</th>
<th>(3) Hemisphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>Right triangle</td>
<td>Hemisphere</td>
</tr>
<tr>
<td><img src="image" alt="Rectangle" /></td>
<td><img src="image" alt="Right triangle" /></td>
<td><img src="image" alt="Hemisphere" /></td>
</tr>
</tbody>
</table>

Solution

1. The cylinder on the right is created.
   \[ \pi \times 5^2 \times 10 = 250\pi \text{ (cm}^3) \]
2. The cone on the right is created.
   \[ \frac{1}{3} \times \pi \times 3^2 \times 6 = 18\pi \text{ (cm}^3) \]
3. The sphere on the right is created.
   \[ \frac{4}{3} \pi \times 5^3 = \frac{500}{3} \pi \text{ (cm}^3) \]

Answer

1. $250\pi \text{ cm}^3$
2. $18\pi \text{ cm}^3$
3. $\frac{500}{3} \pi \text{ cm}^3$

30 Find the volume of the solid that is created when the plane figure below is revolved once around the axis of revolution line $\ell$.

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Rectangle" /></td>
<td><img src="image" alt="Right triangle" /></td>
<td><img src="image" alt="Hemisphere" /></td>
</tr>
</tbody>
</table>

31 Consider the trapezoid ABCD on the right.

\[1\] Answer the questions about the solid that is created when this trapezoid is revolved once with side DC as the axis of revolution.

1. Sketch the solid.
2. Find the volume of the solid.

\[2\] Answer the questions about the solid that is created when this trapezoid is revolved once with side AB as the axis of revolution.

1. Sketch the solid.
2. Find the volume of the solid.
1. Find the volume and surface area of the following solids.

- (1) Cube
- (2) Cylinder
- (3) Right rectangular prism

2. Find the volume and surface area of the following solids.

- (1) Square pyramid
- (2) Rectangular pyramid

3. The shaded solid on the right is created by cutting a cone with a plane that is parallel to its bottom face and taking off the solid having the vertex. Find the volume of this shaded solid.

4. Find the volume and surface area of the following solids.

- (1) Sphere
- (2) Combination of two hemispheres

5. Find the volume of the solid that is created when the plane figure below is revolved once around the axis of revolution line $\ell$.

- (1) L-shaped solid
- (2) Triangular prism
Various ways of looking at solids

Let’s learn the basics 1 Regular polyhedra

Convex polyhedra that have the following two properties are called regular polyhedra.

1 All faces are congruent regular polygons.
2 The number of faces joined at any one vertex is the same.

There are five regular polyhedra, which are shown below.

Regular tetrahedron  Regular hexahedron (Cube)  Regular octahedron  Regular dodecahedron  Regular icosahedron

1 Fill in the blanks to complete the following table about regular polyhedra. This table shows the shape of faces, number of faces, number of edges, number of vertices, number of faces joined at one vertex, and the net of each regular polyhedron. For the net, choose from (i) to (v) below the table.

<table>
<thead>
<tr>
<th>Shape of faces</th>
<th>Regular tetrahedron</th>
<th>Regular hexahedron (Cube)</th>
<th>Regular octahedron</th>
<th>Regular dodecahedron</th>
<th>Regular icosahedron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Number of edges</td>
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<tr>
<td>Number of vertices</td>
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<td></td>
<td></td>
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<tr>
<td>Number of faces joined at one vertex</td>
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<tr>
<td>Net</td>
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</tr>
</tbody>
</table>

(i)  

(ii)  

(iii)  

(iv)  

(v)  

2 A regular hexahedron (cube) has six faces. On each face there is a point where the diagonals of the square intersect. What kind of solid is created by connecting these points?
Let's learn the basics  

Projections

A triangular prism is given in Figure 1. In Figure 2, it is represented by combining a shape seen directly from the front (which is called a front view) and a shape seen directly from the top (which is called a top view). This type of figure is called a projection.

[Note] When drawing projections, edges that can actually be seen are represented as solid lines ——, while edges that cannot be seen are represented as dashed lines ——.

3 Name the solids represented in the following projections.

4 Draw the projections of the following solids as seen from the direction of the arrow.

5 Answer the following questions about the solid shown in the projection on the right.

6 Find the volume of the solid shown in the projection on the right.
Let's learn the basics 3 Using nets

**Question**
The figure shows a cube and its net. Which vertices do (i) and (ii) indicate?

**Solution**
(i) This vertex is on the face that shares edge AD with face ABCD. Besides, it is adjacent to vertex A. Therefore it indicates vertex E.
(ii) This vertex is on the face that shares edge AB with face ABCD. Besides, it is adjacent to vertex B. Therefore it indicates vertex F.

**Answer**
(i) E  (ii) F

7. The figure on the right is the net of the cube shown below. Show the positions of vertices E, F, G, and H on the net.

8. The figure on the right is the net of a regular quadrangular prism. Answer the following questions.

   (1) Find all points that meet point M at a vertex.

   (2) Find all faces that are perpendicular to edge BC and answer using numbers (i) to (vi).

9. When twisting a string around the side face or faces of a solid from point A to point B so that the length of the string is the shortest, as shown in the figures on the right, draw the trace of the string on the net.

10. Suppose you twist a string so that it does not come loose around the cube in Figure 1 on the right. The string starts on point D, crosses edges AB and EF, and reaches point G. Draw the trace of the string on the net in Figure 2.
Let's learn the basics 4 Cutting cubes

**Question** In the cube on the right, points M and N are the midpoints of AB and BC, respectively. When cutting this cube with the following plane, what kind of shape is the cross section?

1. Plane that passes through three points M, N, and F
2. Plane that passes through three points M, N, and E

**Solution**

1. See the figure on the right. The distance between M and F is equal to the distance between N and F, so the cross section is an isosceles triangle with MN = NF.

2. See the figure on the right. Points M, N, E, and G are on the cross section, which is a trapezoid with MN \parallel EG and ME = NG.

**Answer**

1. Isosceles triangle
2. Trapezoid (Isosceles trapezoid)

11 When cutting the cube on the right with the following planes, what kind of shape is each cross section? (Points P to W are the midpoints of each edge.)

- □ (1) Plane that passes through three points B, D, and E
- □ (2) Plane that passes through three points C, D, and E
- □ (3) Plane that passes through three points E, P, and S
- □ (4) Plane that passes through three points A, Q, and G
- □ (5) Plane that passes through three points A, T, and U
- □ (6) Plane that passes through three points F, R, and S
- □ (7) Plane that passes through three points Q, R, and W

12 The figure on the right is a cube whose side is 6 cm. It has a mark on the midpoint of each edge. When you cut the cube with planes that pass through three marks and remove all solids containing the vertices of the cube, answer the following questions about the remaining solid.

- □ (1) How many faces are there on the solid, and what kind of shape are they?
- □ (2) Find the volume of the solid.

13 The figure on the right shows one of the solids made by cutting a cube with a plane. The side of the cube was 6 cm. Name the shape of the cross section, and find the volume of this solid.
1. The figure on the right is a cube whose side is 12 cm. Answer the following questions about the solid whose vertices are the vertices A, C, F, and H of this cube.

☐ (1) What kind of triangle is \( \triangle ACF \)? ☐ (2) Name the solid ACFH.

☐ (3) Find the volume of the solid ACFH.

2. Answer the following questions.

☐ (1) Write suitable numbers in the blanks of the sentences below about how to find the number of edges of a regular dodecahedron. The blanks with the same symbol require the same number.

```
The faces of a regular dodecahedron are \( \text{[ ]} \) regular pentagons, each of which has \( \text{[ ]} \) sides.
So the total number of sides of \( \text{[ ]} \) regular pentagons is \( \text{[ ]} \).
However, since \( \text{[ ]} \) sides of the regular pentagons overlap to form each edge of the regular dodecahedron, the total number of edges of a regular dodecahedron is \( \text{[ ]} \div \text{[ ]} = \text{[ ]} \).
```

☐ (2) Find the number of edges of a regular icosahedron, using the method above.

3. Draw a projection of the solid that is created by revolving the trapezoid on the right once with side DC as the axis of revolution.

4. The figures on the right are the sketch and net of a cuboid. Suppose you twist a string around this cuboid from vertex C to vertex H so that it intersects edges AB and EF, and its length is the shortest. Letting P and Q be the points where the string intersects edges AB and EF respectively, answer the following questions.

☐ (1) Fill in the blanks of the net with suitable letters representing the vertices of this cuboid.

☐ (2) Draw the trace of the string and mark points P and Q on the net.

5. Answer the following questions about the cone in Figure 1 on the right, noting that its bottom face is a circle with radius 1 cm and its generatrix is 3 cm long.

☐ (1) Draw the net of this cone.

☐ (2) Suppose you twist a string around the side face of this cone so that its length is the shortest, as shown in Figure 2 on the right. Letting point A be the end of the string on the bottom face, draw the trace of the string on the net you drew in (1).

Figure 1

Figure 2
1 Positional relationships between lines and planes  The figure on the right shows a solid created by cutting a cuboid with a plane. When AE//BF, answer the following questions.

- (1) How many edges are there that are parallel to edge DC?
- (2) Find all edges that are in skewed positions in relation to edge DC.
- (3) Find a face that is perpendicular to edge EF.
- (4) Is there any face that is parallel to face AEHD?

2 Prisms and cylinders  Find the volume and surface area of the following prisms and cylinder.

- (1) 3 cm 4 cm 5 cm
- (2) 4 cm 5 cm 10 cm 8 cm
- (3) 6 cm 12 cm

3 Cones  Answer the following questions about the cone on the right, noting that its bottom face is a circle with radius 8 cm, its height is 6 cm, and its generatrix is 10 cm long.

- (1) Find the volume of this cone.
- (2) Find the central angle of the sector that makes up the side face.
- (3) Find the surface area of this cone.

4 Spheres  The figure on the right shows one of the solids created by cutting a sphere into 8 equal parts. Find the volume and surface area of this solid.

5 Projections  Name the solid shown in each projection below, and find its volume.

- (1) 
- (2)
1. Find the volume of the solid that is created by revolving the shape below once around the axis of revolution line $\ell$.

\[ \text{Volume} = \pi \times \text{radius}^2 \times \text{height} \]

\[ \text{Volume} = \pi \times 2\text{cm}^2 \times 7\text{cm} \]

\[ \text{Volume} = 46.18\text{cm}^3 \]

2. Suppose you rolled a cone, whose bottom face has a radius of 5 cm, on a plane with its vertex $O$ as the center, as shown in the figure on the right. When the edge of the cone moved on the dotted line and the cone came back to the starting point, it rotated just three times. Answer the following questions about this cone.

\[ \text{Surface Area} = \pi \times r \times l \]

\[ \text{Surface Area} = \pi \times 5\text{cm} \times 5\text{cm} \]

\[ \text{Surface Area} = 78.5\text{cm}^2 \]

3. The figure on the right shows a cube whose side is 6 cm. Point $P$ is on side $DC$. When the volume of quadrangular pyramid $F-PABC$ is $\frac{2}{9}$ of the volume of this cube, find the length of segment $PC$.

\[ \text{Volume of cube} = 6\text{cm}^3 \]

\[ \text{Volume of pyramid} = \frac{1}{3} \times \text{base} \times \text{height} \]

\[ \frac{2}{9} \times 6\text{cm}^3 = \frac{1}{3} \times \text{base} \times \text{height} \]

\[ \text{base} \times \text{height} = 4\text{cm}^3 \]

4. Figure 1 on the right shows a triangular prism with $AB = 3\text{cm}$, $AC = 4\text{cm}$, $\angle BAC = 90^\circ$, $BE = 6\text{cm}$, and $AP = BQ = FR = 2\text{cm}$. Two solids are created by cutting this prism with a plane that passes through three points $P$, $Q$, and $R$. Figure 2 shows the one containing point $D$. Answer the following questions about the solid in Figure 2.

\[ \text{Volume} = \frac{1}{2} \times \text{base} \times \text{height} \]

\[ \text{Volume} = \frac{1}{2} \times 3\text{cm} \times 4\text{cm} \]

\[ \text{Volume} = 6\text{cm}^3 \]
Chapter 7 Organizing and making use of data

Key points of study

1. Frequency distribution
   (1) Each section to organize data items is called a class. Each class has an interval, and the number of data items in each class is called the frequency of that class. A table that shows frequencies according to class is called a frequency distribution table.
   (2) The difference between the largest value and the smallest value in a data set is called the range of the distribution.

2. Histograms, Frequency distribution polygons
   (1) Frequency distribution can be expressed by a graph of a series of rectangles whose width represents each class and whose height represents frequency. Such a graph is called a histogram (or columnar graph).
   (2) Marking the midpoint of the top side of each rectangle in a histogram and connecting the marks gives you a line graph. This graph is called a frequency distribution polygon (or frequency distribution line graph).

3. Relative frequency
   In a distribution table, \( \frac{\text{Frequency of each class}}{\text{Total frequency}} \) is called the relative frequency.

4. Representative value
   A value that represents the entire data set is called a representative value. The mean, the median, and the mode, etc., are used as representative values.

5. Mean
   (1) In a frequency distribution table, the middle value of a class is called the class value.
   (2) How to find the mean \( \frac{\text{Sum of (Class value) \times (Frequency)}}{\text{(Total frequency)}} \)
   (3) The mean can also be found using the tentative mean, as follows:
      ① Let the tentative mean be the class value that seems to be close to the mean, and find the value for (Class value) - (Tentative value) in each class.
      ② Multiply each value you get in ① by its frequency and add up the products.
      ③ Divide the result of ② by the total frequency, and add the quotient to the tentative mean.

6. Median, Mode
   (1) When arranging the values of a data set in size order, the middle value is called the median. If there are an even number of data items, the median is the mean of the two middle values.
   (2) The value that appears most frequently in a set of data is called the mode. In a frequency distribution table, the mode is the class value for the value with the highest frequency.

7. Approximate values
   (1) A value close to the true value, such as a measured value or a rounded value, is called an approximate value.
   (2) The difference between an approximate value and a true value is called an error.
   (3) When a number represents an approximate value, reliable digits are called significant figures. You can make it clear which digits are significant figures by expressing the number as the product of a decimal with a one-digit integer portion and 10 multiplied to some power.
Let's learn the basics  I  Frequency distribution tables

1. Each section to organize data in a frequency distribution table is called a class. Each class has an interval, and the number of data items in each class is called the frequency of that class.

2. The difference between the largest value and the smallest value in a data set is called the range of the distribution.

\[(\text{Range}) = (\text{Largest value}) - (\text{Smallest value})\]

---

**Question**  The frequency distribution table on the right shows the height measurement records of 40 first-year boys at junior high school A. Answer the following questions.

1. How many centimeters is the interval of each class?  
2. Which class has the highest frequency?  
3. What is the percentage of boys whose heights are at least 160 cm?

**Solution**  
1. Each class includes two heights with a difference of 5 cm, which is the interval of each class.
2. Look for the class with the highest frequency, which is 14.
3. 13 (\(=8+5\)) boys are at least 160 cm tall, so the percentage is \(13 \times 100 = 32.5\) (%)

**Answer**  
1. 5 cm  
2. 155-160 cm class [at least 155 cm and less than 160 cm]  
3. 32.5%

---

1. The frequency distribution table on the right shows the weight measurement records of 40 first-year boys at a junior high school. Answer the following questions.

- (1) How many kilograms is the interval of each class?  
- (2) Which class has the highest frequency?  
- (3) What is the percentage of boys who weigh less than 45 kg?

---

2. The set of data below shows the handball-throw records of 20 girls in a class. Answer the following questions.

- 17 13 19 10 12 17 24 15 20 18  
  23 18 12 15 17 11 14 16 19 22 (Unit: m)

- (1) Find the range of the distribution.  
- (2) Fill in the blanks (i) to (iii) in the frequency distribution table on the right.  
- (3) What is the percentage of girls who threw a ball at least 20 m?
Let's learn the basics  

2. Histograms

1. Frequency distribution can be expressed by a graph of a series of rectangles whose width represents each class and whose height represents frequency. Such a graph is called a histogram (or columnar graph).

2. Marking the midpoint of the top side of each rectangle in a histogram and connecting the marks gives you a line graph. This graph is called a frequency distribution polygon (or frequency distribution line graph).

---

**Question:** The histogram on the right shows the weight measurement records of boys in a class. Answer the following questions.

1. How many boys are there in this class?
2. To which class do the fifth heaviest boy belong?
3. Draw a frequency distribution polygon on the graph on the right.

**Solution:**

1. The sum of the frequencies of all classes is 2+4+8+5+3+1=23

2. The frequency of the 60-65 kg class is 1, and that of the 55-60 kg class is 3.

   So, there are 4 boys who weigh at least 55 kg. Therefore, the fifth heaviest boy belongs to the 50-55 kg class.

3. To draw a frequency distribution polygon, connect the midpoint of the top side of each rectangle in a histogram, and extend the line beyond each end by marking a point that represents an imaginary class with a frequency of 0.

**Answer**

1. 23 boys  
2. the 50-55 kg class [At least 50 kg and less than 55 kg]
3. See the graph on the right.

---

3. The histogram on the right shows the grip measurement records of boys in a class. Answer the following questions.

   □1) How many boys are there in this class?
   □2) To which class does the boy having the eighth strongest grip belong?
   □3) What percentage of boys have a grip of less than 30 kg?
   □4) Draw a frequency distribution polygon on the graph on the right.

---

4. The table on the right shows the amount of time 40 students in a class take to go to school. Draw a histogram to express this frequency distribution.
Let's learn the basics: Relative frequency

The frequency of each class divided by the total frequency is called the relative frequency of that class.

\[(\text{Relative frequency}) = \frac{\text{(Frequency of each class)}}{\text{(Total frequency)}}\]

**Question** The frequency and relative frequency distribution table on the right show the handball-throw records of 40 first-year girls of a junior high school. Fill in blanks (i) to (iv) with suitable numbers.

**Solution**

(i) \(\frac{8}{40} = 0.20\), (ii) \(\frac{14}{40} = 0.35\). Letting \(x\) be \(x\), \(\frac{x}{40} = 0.30\), \(x = 12\). Letting \(y\) be \(\frac{y}{40} = 0.10\), \(y = 4\).

**Answer** (i) \(0.20\), (ii) \(0.35\), (iii) \(12\), (iv) \(4\).

*Values of decimals in a data set are often given the same number of decimal places. For example, \(\frac{8}{40}\) is usually expressed as 0.2 in decimal form, but if the other data items in the set have two decimal places, 0.2 is expressed as 0.20.*

5 The table on the right shows the heights of first-year girls at a junior high school. Fill in the blanks (i) to (iv) with suitable numbers.

<table>
<thead>
<tr>
<th>Class(cm)</th>
<th>Frequency</th>
<th>Relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least 140</td>
<td>12</td>
<td>(i)</td>
</tr>
<tr>
<td>145~150</td>
<td>21</td>
<td>0.14</td>
</tr>
<tr>
<td>150~155</td>
<td>36</td>
<td>(ii)</td>
</tr>
<tr>
<td>155~160</td>
<td>(iii)</td>
<td>0.20</td>
</tr>
<tr>
<td>160~165</td>
<td>27</td>
<td>0.18</td>
</tr>
<tr>
<td>165~170</td>
<td>(iv)</td>
<td>0.06</td>
</tr>
<tr>
<td>Total</td>
<td>150</td>
<td>1.00</td>
</tr>
</tbody>
</table>

6 Table 1 shows the long-jump records of 50 first-year boys and 60 second-year boys at a junior high school. Table 2 shows the distribution of relative frequency of each class. Graph 3 shows the relative frequency distribution polygon about the first-year boys.

<table>
<thead>
<tr>
<th>Class(cm)</th>
<th>Frequency</th>
<th>Relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least 260</td>
<td>10</td>
<td>0.20</td>
</tr>
<tr>
<td>300~340</td>
<td>14</td>
<td>0.28</td>
</tr>
<tr>
<td>340~380</td>
<td>16</td>
<td>0.32</td>
</tr>
<tr>
<td>380~420</td>
<td>2</td>
<td>0.16</td>
</tr>
<tr>
<td>420~460</td>
<td>2</td>
<td>0.04</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Class(cm)</th>
<th>Frequency</th>
<th>Relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least 260</td>
<td>10</td>
<td>0.20</td>
</tr>
<tr>
<td>300~340</td>
<td>14</td>
<td>0.28</td>
</tr>
<tr>
<td>340~380</td>
<td>16</td>
<td>0.32</td>
</tr>
<tr>
<td>380~420</td>
<td>2</td>
<td>0.16</td>
</tr>
<tr>
<td>420~460</td>
<td>2</td>
<td>0.04</td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
<td>1.00</td>
</tr>
</tbody>
</table>

1) Fill in the blanks of Table 2 with the suitable relative frequencies about the second-year boys.

2) Draw the relative frequency distribution polygon about the second-year boys on Graph 3.

3) Which jumped relatively longer, the first-year boys or the second-year boys?
Let's learn the basics  4  Mean

1. In a frequency distribution table, the middle value of a class is called the **class value**.
2. The sum of all individual data values divided by the number of data items (total frequency) is called the **mean**.

To find the mean using a frequency distribution table, the following formula is used.

\[
(Mean) = \frac{\text{Sum of (Class value) \times (Frequency)}}{\text{(Total frequency)}}
\]

**Question**  The table on the right shows the grip measurement records of 46 first-year boys at a junior high school. Answer the following questions.

1. Fill in the blanks (i) to (iii) with suitable numbers.
2. Find the mean and round it to one decimal place.

**Solution**

1. (i) \( \frac{30 + 35}{2} = 32.5 \)
2. (ii) \( 32.5 \times 14 = 455.0 \)
3. (iii) \( 157.5 + 220.0 + 455.0 + 487.5 + 170.0 = 1490.0 \)

\( (2) \) Dividing the value of (iii) by the total frequency 46 gives you \( 1490.0 \div 46 = 32.39 \ldots \rightarrow 32.4 \text{ kg} \)

**Answer**

1. (1) (i) \( 32.5 \), (ii) \( 455.0 \), (iii) \( 1490.0 \)  (2) 32.4 kg

**Question**  The table on the right shows the distance 20 girls in a class travel from their houses to the school. Answer the following questions.

1. Fill in the blanks with suitable numbers.
2. Find the mean travel distance and round it to one decimal place.

**Answer**

1. (1) (i) \( 32.5 \), (ii) \( 455.0 \), (iii) \( 1490.0 \)  (2) 32.4 kg

**Question**  The table on the right shows the height measurement records of students in a class. Answer the following questions.

1. Fill in the blanks with suitable numbers.
2. Find their mean height and round it to one decimal place.
The mean can sometimes be calculated using the tentative mean.

\[
\text{(Mean)} = \frac{\text{Sum of } \{(\text{Class value}) - \text{(Tentative mean)}\} \times \text{(Frequency)}}{\text{(Total frequency)}}
\]

The tentative mean should be the class value that seems to be close to the mean.

**Question** The table on the right shows the vertical jump records of 20 first-year girls at a junior high school. When calculating the mean by using the tentative mean 45 cm, answer the following questions.

1. Fill in the blanks (i) to (iii) with suitable numbers.
2. Find the mean.

**Solution**

1. (i) \(-20\), (ii) \(-50\), (iii) \(-70\)
2. \(45 + \frac{-70}{20} = 45 - 3.5 = 41.5\) cm

**Answer**

1. (i) \(-20\), (ii) \(-50\), (iii) \(-70\)  
2. 41.5 cm

**9** The table below shows the 50-meter sprint records of 80 first-year girls at a junior high school. When calculating the mean by using the tentative mean 8.4 seconds, answer the following questions.

<table>
<thead>
<tr>
<th>Class(second)</th>
<th>Class value(second)</th>
<th>Class value-8.4(second)</th>
<th>Frequency</th>
<th>(Class value-8.4) × (Frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least less than 7.4~7.8</td>
<td>7.6</td>
<td>-0.8</td>
<td>9</td>
<td>-7.2</td>
</tr>
<tr>
<td>7.8~8.2</td>
<td>8.0</td>
<td>(ii)</td>
<td>21</td>
<td>-8.4</td>
</tr>
<tr>
<td>8.2~8.6</td>
<td>8.4</td>
<td>0</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>8.6~9.0</td>
<td>(i)</td>
<td>0.4</td>
<td>(iv)</td>
<td>7.2</td>
</tr>
<tr>
<td>9.0~9.4</td>
<td>9.2</td>
<td>0.8</td>
<td>(v)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>80</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Class(minute)</th>
<th>Class value(minute)</th>
<th>Class value-105(minute)</th>
<th>Frequency</th>
<th>(Class value-105) × (Frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least less than 30~60</td>
<td>60</td>
<td>-35</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>60~90</td>
<td>90</td>
<td>-25</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>90~120</td>
<td>120</td>
<td>-55</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>120~150</td>
<td>150</td>
<td>-85</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>150~180</td>
<td>180</td>
<td>-115</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>
Let's learn the basics

### Median, Mode

1. When arranging the values of a data set in size order, the middle value is called the **median**. If there are an even number of data items, the median is the mean of the two middle values.

2. The value that appears most frequently in a set of data, or the class value for the value with the highest frequency in a frequency distribution table, is called the **mode**.

### Question 1

The set of data on the right shows the weight of 10 members of the sumo club at a school. Find the median of their weight.

<table>
<thead>
<tr>
<th>Weight (kg)</th>
<th>80</th>
<th>87</th>
<th>94</th>
<th>77</th>
<th>90</th>
<th>98</th>
<th>93</th>
<th>84</th>
<th>83</th>
<th>91</th>
</tr>
</thead>
</table>

**Solution**

The weights, can be arranged from light to heavy as follows:

77, 80, 83, 84, 87, 90, 91, 93, 94, 98

There are an even number of data items.

So, the median is the mean of the two middle values 87 and 90, which is \( \frac{87 + 90}{2} = 88.5 \) (kg).

**Answer**

88.5 kg

### Question 2

The table on the right shows the quiz scores of students in a class. Answer the following questions.

1. Find the median.
2. Find the mode.

### Solution

1. The number of students who got at most 1 point is 2+9=11. The number of those who got at most 2 points is 11+11=22. Therefore, the students who got the 20th and 21st lowest scores (two middle data items) both got 2 points.

2. The highest frequency is 13, and the value with the highest frequency is 3 points.

**Answer**

1. 2 points
2. 3 points

### Question 3

The set of data on the right shows the height of 9 members of the volleyball club at a school. Find the median of their height.

169 178 181 172 175

180 176 171 183

**Solution**

The heights, can be arranged from light to heavy as follows:

169, 171, 172, 176, 178, 181, 183, 184, 185

So, the median is the mean of the two middle values 178 and 181, which is \( \frac{178 + 181}{2} = 179.5 \) (kg).

**Answer**

179.5 cm

### Question 4

The set of data on the right shows the height of 12 apartment buildings. Find the median of their height.

48 32 40 50 34 57

28 62 43 37 55 36

**Solution**

The heights, can be arranged from light to heavy as follows:

28, 32, 34, 37, 40, 43, 50, 55, 57, 62

So, the median is the mean of the two middle values 40 and 43, which is \( \frac{40 + 43}{2} = 41.5 \) (cm).

**Answer**

41.5 cm

### Question 5

The table on the right shows the number of comic books read by first-year students of a junior high school in a week. Answer the following questions.

1. Find the median.
2. Find the mode.

### Solution

The number of books, can be arranged from light to heavy as follows:

1, 2, 3, 4, 5, 6, 7, 8

The highest frequency is 14, and the value with the highest frequency is 3.

**Answer**

1. 3 books
2. 3 books

### Question 6

The frequency distribution table on the right shows the weight of the first-year boys at a junior high school. Answer the following questions.

1. To which class does the median belong?
2. Find the mode.

### Solution

The weight, can be arranged from light to heavy as follows:

35, 40, 45, 50, 55

The median is in the class 40-45.

**Answer**

1. 40-45 kg
2. 45+50 kg

**Table**

<table>
<thead>
<tr>
<th>Class (kg)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least less than</td>
<td></td>
</tr>
<tr>
<td>35-40</td>
<td>4</td>
</tr>
<tr>
<td>40-45</td>
<td>10</td>
</tr>
<tr>
<td>45-50</td>
<td>12</td>
</tr>
<tr>
<td>50-55</td>
<td>9</td>
</tr>
<tr>
<td>55-60</td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
</tr>
</tbody>
</table>
Let's learn the basics

Approximate values and errors, Significant figures

1. A value close to the true value, such as a measured value or a rounded value, is called an approximate value.

2. The difference between an approximate value and a true value is called an error.

   \[(\text{Error}) = (\text{Approximate value}) - (\text{True value})\]

3. When a number represents an approximate value, reliable digits are called significant figures. You can make it clear which digits are significant figures by expressing the number as the product of a decimal with a one-digit integer portion and 10 multiplied to some power.

---

**Question 1**

When a decimal \(a\) rounded to the nearest whole number is 38, answer the following questions.

1. Express the range of \(a\) using inequality signs.
2. Find the greatest absolute value of error.

**Solution**

1. The value of \(a\) should be at least 37.5 and less than 38.5.
2. \(\text{Error} = (\text{Approximate value}) - (\text{True value})\). Therefore, the greatest absolute value of error is 38 – 37.5 = 0.5.

**Answer**

1. \(37.5 \leq a < 38.5\)  
2. 0.5

---

**Question 2**

When 580 g is an approximate value measured to the tens place, express it so that the significant figures are clear.

**Solution**

In the measured value 580 g, the hundreds digit 5 and the tens digit 8 are reliable, while the ones digit 0 is not. The significant figures are 5 and 8, so 580 is expressed as \(5.8 \times 10^2\) g.

**Answer**

\(5.8 \times 10^2\) g

---

15 When a number \(a\) rounded to one decimal place is 6.3, answer the following questions.

1. Express the range of \(a\) using inequality signs.
2. Find the greatest absolute value of error.

16 Express the following measured values so that the significant figures are clear.

1. 630 cm as measured to the tens place
2. 490 cm as measured to the ones place
3. 7200 g as measured to the hundreds place
4. 3100 g as measured to the tens place

17 To which place were the measured values below measured?

1. \(5.7 \times 10^2\) m
2. \(7.3 \times 10^3\) km
3. \(1.50 \times 10^2\) g
4. \(1.02 \times 10^4\) kg

18 Answer the following questions.

1. Express 3500 g as an approximate value with a two-digit significant figure.
2. Express 64000 m² as an approximate value with a two-digit significant figure.
3. Express 8270 cm as an approximate value with a three-digit significant figure.
4. Express 19000 mg as an approximate value with a three-digit significant figure.
1 Frequency distribution tables  The frequency distribution table on the right shows the chest circumference of 20 boys in a class. Answer the following questions.

□ (1) How many centimeters is the interval of each class?

□ (2) Which class has the highest frequency?

□ (3) Find the relative frequency of the 85-90 cm class.

<table>
<thead>
<tr>
<th>Class (cm)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least</td>
<td>less than</td>
</tr>
<tr>
<td>65~70</td>
<td>2</td>
</tr>
<tr>
<td>70~75</td>
<td>6</td>
</tr>
<tr>
<td>75~80</td>
<td>7</td>
</tr>
<tr>
<td>80~85</td>
<td>4</td>
</tr>
<tr>
<td>85~90</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
</tr>
</tbody>
</table>

2 Histograms The figure on the right is a histogram that shows the weight of first-year students of a class at a junior high school. Answer the following questions.

□ (1) Find the number of students of this class.

□ (2) Find the class value of the class to which the 12th heaviest student belongs.

□ (3) Find the relative frequency of the 45-50 kg class and round it to the second decimal place.

3 Representative values The table on the right shows the weight measurement records of students in a class. Answer the following questions.

□ (1) Find the mean weight by rounding it to the first decimal place.

□ (2) To which class does the median belong?

□ (3) Find the mode.

<table>
<thead>
<tr>
<th>Class (kg)</th>
<th>Class value (kg)</th>
<th>Frequency</th>
<th>Class value × (Frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least</td>
<td>less than</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35~40</td>
<td>37.5</td>
<td>3</td>
<td>112.5</td>
</tr>
<tr>
<td>40~45</td>
<td>42.5</td>
<td>11</td>
<td>467.5</td>
</tr>
<tr>
<td>45~50</td>
<td>47.5</td>
<td>14</td>
<td>665.0</td>
</tr>
<tr>
<td>50~55</td>
<td>52.5</td>
<td>8</td>
<td>420.0</td>
</tr>
<tr>
<td>55~60</td>
<td>57.5</td>
<td>4</td>
<td>230.0</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>40</td>
<td>1895.0</td>
</tr>
</tbody>
</table>

4 Significant figures The mean distance between the Sun and the planet Mercury is said to be about 57900000 km. Express this approximate number as a three-digit significant figure.
1. The table on the right shows the height of 20 tennis club members at a junior high school.
   □(1) Find the values for \(a\), \(b\), \(c\), and \(d\).
   □(2) Find their mean height.
   □(3) Later, this club was joined by some members belonging to the 170–175 cm class, and the table was revised. In the new table, the mean height increased by 2.0 cm. How many new members joined the club?

<table>
<thead>
<tr>
<th>Class (cm)</th>
<th>Class value (cm)</th>
<th>Frequency</th>
<th>((\text{Class value}) \times \text{(Frequency)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>145~150</td>
<td>147.5</td>
<td>(a)</td>
<td>590.0</td>
</tr>
<tr>
<td>150~155</td>
<td>152.5</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
<td>155~160</td>
<td>157.5</td>
<td>4</td>
<td>630.0</td>
</tr>
<tr>
<td>160~165</td>
<td>162.5</td>
<td>5</td>
<td>812.5</td>
</tr>
<tr>
<td>165~170</td>
<td>167.5</td>
<td>3</td>
<td>502.5</td>
</tr>
<tr>
<td>170~175</td>
<td>172.5</td>
<td>2</td>
<td>345.0</td>
</tr>
<tr>
<td>175~180</td>
<td>177.5</td>
<td>1</td>
<td>177.5</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>20</td>
<td>(d)</td>
</tr>
</tbody>
</table>

2. The table on the right shows the scores at a Chinese character test given to all first-year boys and girls of a junior high school. The girls’ mean score was 5.56 points, and the total mean score was 5.48 points.
   □(1) Find the boys’ mean score.
   □(2) Find the median and mode of the boys’ score.
   □(3) Find boys’ score range and girls’ score range.
   □(4) Find the value for \(x\) and \(y\) in the table.

<table>
<thead>
<tr>
<th>Score</th>
<th>Boys</th>
<th>Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>(y)</td>
</tr>
<tr>
<td>6</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>(x)</td>
</tr>
</tbody>
</table>

3. An examination was conducted at a rice paddy to study how many husks each rice stalk had. Table 1 on the right shows the number of husks on each of 20 stalks. The numbers in Table 1 were organized in Table 2, a frequency distribution table. Letters \(x\) and \(y\) in Table 1 represent the number of husks; letters \(m\) and \(n\) in Table 2 represent the number of stalks.
   □(1) Find the relative frequency of the 78–80 husks class.
   □(2) Find the values for \(m\) and \(n\).
   □(3) When \(x–y=5\), find the values for \(x\) and \(y\).

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>82</td>
<td>77</td>
</tr>
<tr>
<td>79</td>
<td>80</td>
</tr>
<tr>
<td>76</td>
<td>86</td>
</tr>
<tr>
<td>(y)</td>
<td>81</td>
</tr>
<tr>
<td>81</td>
<td>84</td>
</tr>
<tr>
<td>80</td>
<td>79</td>
</tr>
<tr>
<td>77</td>
<td>74</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
</tr>
<tr>
<td>Class (Number of husks)</td>
<td>Frequency (Number of stalks)</td>
</tr>
<tr>
<td>at least less than</td>
<td></td>
</tr>
<tr>
<td>72~74</td>
<td>(m)</td>
</tr>
<tr>
<td>75~77</td>
<td>(n)</td>
</tr>
<tr>
<td>78~80</td>
<td>7</td>
</tr>
<tr>
<td>81~83</td>
<td>5</td>
</tr>
<tr>
<td>84~86</td>
<td>2</td>
</tr>
</tbody>
</table>

4. Approximate values are sometimes expressed as \(a \times \frac{1}{10^n}\) (\(a\) is a decimal with a one-digit integer portion and \(n\) is a natural number). For example, 0.054 g, an approximate value with a two-digit significant figure, is expressed as \(5.4 \times \frac{1}{10^2}\) g. In this way, express 0.00033 m as an approximate value with a two-digit significant figure, in the form of \(a \times \frac{1}{10^2}\).
Let's review

Signed numbers

1 Express the relationships between the size of the following pairs of numbers using inequality signs.

-1) \(-4.3, -4.7\)
-2) \(-2, \frac{8}{3}, -2.5\)

2 Answer the following questions.

-1) Find the absolute value of \(-0.07\).
-2) Find all integers whose absolute value is at least 5 and at most 8.

Addition and subtraction

3 Calculate.

-1) \((-9) + 3\)
-2) \(6.6 + (-2.9)\)
-3) \(-59 - (-61)\)
-4) \(\frac{2}{3} - \frac{3}{4}\)

4 Calculate.

-1) \((-11) - (-15) + (-6)\)
-2) \(-5 + 15 - (-12)\)
-3) \(8 - 17 + 10 - 6\)
-4) \(-4.2 + 5.3 - (-1.6) - 2\)
-5) \(-\frac{1}{2} + \frac{2}{3} - \frac{5}{6}\)

Multiplication and division

5 Calculate.

-1) \((-4) \times (-6)\)
-2) \(20 \times (-8)\)
-3) \((-36) \div (-4)\)
-4) \((-5.4) \div 0.9\)
-5) \((-8) \div (-12) \times (-15)\)
-6) \(6 \times (-3)^2 \div 27\)

Calculations with the four arithmetic calculations

6 Calculate.

-1) \(15 - 7 \times (-8)\)
-2) \(21 \div (2 - 5) + 5\)
-3) \(\frac{3}{5} + \frac{1}{6} \times \left(\frac{-4}{5}\right)\)
-4) \((-3)^2 \times 2 - 4^2 \div 8\)
-5) \(\left(\frac{-1}{2}\right)^2 \div \left(\frac{3}{4} - 3 \div \frac{2}{3}\right)\)

Using signed numbers

7 The table on the right shows the results of subtracting F's score on an English test from each score of students A to F. If F's score is 71 points, what is the average score of these 6 students?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>+5</td>
</tr>
<tr>
<td>B</td>
<td>-9</td>
</tr>
<tr>
<td>C</td>
<td>+8</td>
</tr>
<tr>
<td>D</td>
<td>-3</td>
</tr>
<tr>
<td>E</td>
<td>-13</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
</tr>
</tbody>
</table>

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Let's review  ②  Algebraic expressions

[How to write algebraic expressions]
1 Write the following expressions without using the symbols × or ÷.
  □(1) \( x \times 3 \div y \)
  □(2) \( 5 \div (a+b) \times c \)
  □(3) \( x \div (-2) + 4 \times y \)
  □(4) \( 9 \div (a+b) - c \)

[How to express quantities]
2 Write an expression to indicate the following quantities.
  □(1) The average weight of three apples that weigh \( a \) kg, \( b \) kg, and \( c \) kg.
  □(2) The circumference of a circle whose radius is 6 cm (Use \( \pi \) for the ratio of the circumference of a circle to its diameter.)
  □(3) The amount of salt contained in the salt water made by mixing \( x \) g of 5% salt water and \( y \) g of 10% salt water.

3 Write expressions to indicate the following about the cuboid on the right.
  □(1) Volume
  □(2) Surface area
    (Sum of the areas of all faces)

[The value of expressions]
4 Answer the following questions.
  □(1) When \( x = 5 \), find the value for \( 3x - 7 \).
  □(2) When \( a = -4 \), find the value for \( 2a^2 \).

[Calculation with linear expressions]
5 Calculate.
  □(1) \( 3a + 5 + 2 + 5a \)
  □(2) \( (5x - 3) + (-9x - 2) \)
  □(3) \( -(a+6) - (8a+1) \)
  □(4) \( 7(5a+4) - 2(a-3) \)
  □(5) \( 6 \times \frac{x-3}{2} + 8 \times \frac{2x+1}{4} \)
  □(6) \( \frac{7x+10}{3} - \frac{2x+5}{9} \)

[Using algebraic expressions]
6 Answer the following questions.
  □(1) The admission fee for adults to an art museum is \( a \) yen, and that for junior high school students is 350 yen less. How much is the total admission fee for 3 adults and 4 junior high school students?
  □(2) Find the area of the shaded part of the figure on the right. Use \( \pi \) for the ratio of the circumference of a circle to its diameter.
  □(3) Three times \( x \) plus 2 is 20. Write an equality to indicate this relationship.
  □(4) It takes at least an hour to travel 10 km, if you first walk \( x \) km at a speed of 6 km per hour and then run the remaining at 12 km per hour. Write an inequality to indicate this relationship.
Let's review

Equations

(How to solve equations)

1. Solve the following equations.
   □ (1) \( x - 3 = 7 \)
   □ (2) \( 4x = -20 \)
   □ (3) \( 3x - 6 = x \)
   □ (4) \( 7x + 10 = 5x - 2 \)
   □ (5) \( 10 + 3x = 8x - 15 \)
   □ (6) \( 7 - 2(x - 1) = -3x + 5 \)
   □ (7) \( 3(x + 1) - 2(3x - 2) = 4 \)

2. Solve the following equations.
   □ (1) \( -1.7 + 0.8x = x - 0.3 \)
   □ (2) \( 1.25x - 0.1 = 2x + 2.15 \)
   □ (3) \( \frac{x + 1}{3} + \frac{x - 4}{2} = 5 \)
   □ (4) \( -4 + \frac{1}{5}x = -\frac{1}{2}x + 3 \)
   □ (5) \( \frac{1}{5}(x - 3) - \frac{7}{10}x = -\frac{1}{10} \)
   □ (6) \( x : 3 = 0.2 : 0.6 \)
   □ (7) \( (x + 2) : (x + 5) = 3 : 4 \)

(Using equations)

3. When the solution to the equation in terms of \( x \), \( ax + 5(x - a) = 7 \), is \( x = 2 \), find the value for \( a \).

4. You paid 750 yen for 12 desserts, which were some 80-yen okagi and some 50-yen manju. Answer the following questions.
   □ (1) Letting \( x \) be the number of okagi you bought, you can write the following equation. Fill in the blank with a suitable expression.
     \[ 80x + \text{[blank]} = 750 \]  
   □ (2) Solve the equation (1) and find how many okagi and how many manju you bought.

5. A person walked from point P to point Q and back. She walked to point Q at a speed of 60 m per minute. Then she walked back to point P at 50 m per minute, so it took 4 more minutes. How many meters is the distance between point P and point Q?

6. You fixed the tag price of an article, expecting a profit of 20% of its cost price. If you sell it at a 10% discount, you will get a profit of 20 yen. Find the cost price of this article.

7. Rectangle ABCD is given on the right.
   Point P moves from D to C on side CD at a speed of 2 cm per second. How many seconds after point P starts from D does the area of \( \triangle APD \) become 96 cm\(^2\)?
Let's review 4 Proportion and inverse proportion

(Function, Proportion and inverse proportion)

1 Write an expression to indicate the relationships between $x$ and $y$ in the following. If $y$ is proportional or inversely proportional to $x$, what is the constant of proportion?

- (1) The area of a triangle with an $x$-cm base and a 10-cm height is $y$ cm².
- (2) The circumference of a rectangle $x$ cm long and $y$ cm wide is 30 cm.
- (3) It takes $y$ minutes to fill up a 600-L bathtub by pouring in water at a rate of $x$ L per minute.

2 When $x$ and $y$ in the tables below have the relationships shown in (1) and (2), express $y$ in terms of $x$. Then fill in the blanks.

- (1) Proportion
  
<table>
<thead>
<tr>
<th>$x$</th>
<th>3</th>
<th>9</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

- (2) Inverse proportion
  
<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>6</td>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3 Answer the following questions.

- (1) $y$ is proportional to $x$. When $x = -2$, $y = 10$. Find the value for $y$ when $x = 3$.
- (2) $y$ is inversely proportional to $x$. When $x = -3$, $y = 8$. Find the value for $x$ when $y = 4$.

4 Find the coordinates of the following points.

- (1) Reflection of $(-7, 4)$ about the $y$-axis
- (2) Midpoint between $(-2, -3)$ and $(8, 1)$

5 Graph the following.

- (1) $y = \frac{-2}{3}x$
- (2) $y = \frac{12}{x}$

6 In the figure on the right, $\odot$ and $\otimes$ are graphs of proportion and inverse proportion, respectively. The coordinates of one of the intersections of graphs $\odot$ and $\otimes$ are $(2, 4)$.

- (1) Write the expressions of graphs $\odot$ and $\otimes$.
- (2) Find the values for $p$ and $q$.

7 Answer the following questions.

- (1) Find the area of $\triangle ABC$ with vertices $A(-1, 3), B(-4, 2)$, and $C(-2, 4)$.
- (2) When parallelogram $ABCD$ has vertices $A(-3, 4), B(-1, -5)$, and $C(4, -4)$, find the coordinates of vertex $D$.
- (3) In the figure on the right, quadrilateral $ABCD$ is a square and the graph of $y = \frac{1}{3}x$ passes through point $A$.

  When the $x$-coordinate of $C$ is 8, find the coordinates of $A$.  

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Let's review

1. Lines $\ell$, $m$, and $n$ and points $A$ to $E$ are given in the figure on the right. Answer the following questions.
   - (1) Express the positional relationship between line $\ell$ and line $m$ using a symbol.
   - (2) Express the positional relationship between line $m$ and line $n$ using a symbol.
   - (3) Which point is at the same distance from line $\ell$ as point $A$?

2. Fill in the blanks below.
   - (1) In the figure on the right, segment $AB$ is called a ___.
     The part shown with the thick line is called an ___, which is written as [___] with a symbol.
   - (2) In the figure on the right, line $\ell$ is tangent to circle $O$ at point $P$. The positional relationship between line $\ell$ and radius $OP$ is expressed as $\ell$ [___] $OP$ with a symbol.

3. In the figure on the right, $Q$ and $R$ are the tangent points where the tangents drawn from point $P$ are tangent to circle $O$. When $\angle QPR = 38^\circ$, find the size of $\angle QOR$.

4. In the figure on the right, the inside of square $ABCD$ is divided into 8 congruent triangles. Answer the following questions.
   - (1) Which triangle does $\triangle DGO$ overlap when it is translated?
   - (2) Which triangle does $\triangle OHD$ overlap when it is rotated 90° clockwise around the center of rotation $O$?
   - (3) Which triangle does $\triangle AEO$ overlap when it is reflected over the axis of reflection $BD$?

5. Construct the following figures.
   - (1) Perpendicular bisector of segment $AB$
   - (2) Bisector of $\angle AOB$
6 Construct the following figures.
   □(1) Line that is perpendicular to line $\ell$ and passes through point $P$
   ![Diagram 1]

   □(2) Line that is perpendicular to line $\ell$ and passes through point $P$
   ![Diagram 2]

7 Construct the following figures.
   □(1) Isosceles right triangle $ABC$ with segment $AB$ as a side and $\angle ABC = 90^\circ$
   ![Diagram 3]

   □(2) Circle with segment $AB$ as a chord and its center on line $\ell$
   ![Diagram 4]

8 Construct the following figures.
   □(1) Figure that has line symmetry with side $BC$ as the axis of symmetry
   ![Diagram 5]

   □(2) Center of rotation when segment $CD$ is rotated from segment $AB$ (points $A$ and $B$ correspond to points $C$ and $D$, respectively)
   ![Diagram 6]

9 Find the arc length and area of the following sectors.
   □(1) Sector with radius 8 cm and central angle $45^\circ$
   □(2) Sector with radius 12 cm and central angle $210^\circ$

10 Find the central angle of a sector with radius 6 cm and arc length $5\pi$ cm.

11 The figures below are combinations of sectors and a rectangle or triangle. Find the area of the shaded part.
   □(1)
   ![Diagram 7]
   □(2)
   ![Diagram 8]
Let's review

[Positional relationships]
1. In the quadrangular prism ABCD-EFGH shown below, \( \angle BAD = \angle ADC = 90^\circ \). Answer the following questions.

- 1) Find all edges that are parallel to face AEHD.
- 2) Find all edges that are in skewed positions in relation to edge BC.

2. Mark a circle (\( \bigcirc \)) if each of the following statements about the positional relationships in space is always right, and mark a cross (\( \times \)) if it isn't.
   - If two planes are both parallel to a line, they are parallel to each other.
   - Suppose there are two parallel lines and a plane. If the plane is perpendicular to one line, it is perpendicular to the other line, too.

[Prisms, cylinders, pyramids, and cones, Spheres, Solids of revolution]
3. Find the volume of the solids below.

- 1) \( \text{Volume} = \frac{1}{3} \times \text{base} \times \text{height} = \frac{1}{3} \times 2 \times 4 \times \frac{1}{2} = 4 \) cm³.
- 2) \( \text{Volume} = \frac{1}{3} \times \pi \times (\frac{8}{2})^2 \times 8 = \frac{1}{3} \times \pi \times 16 \times 8 = \frac{128}{3} \pi \) cm³.

4. Find the volume and surface area of the solids below.

- 1) \( \text{Volume} = \pi \times \text{radius}^2 \times \text{height} = \pi \times 5^2 \times 6 = 150\pi \) cm³.
- 2) \( \text{Volume} = \frac{1}{3} \times \pi \times (\frac{12}{2})^2 \times 15 = 360\pi \) cm³.

5. Find the volume and surface area of the hemisphere on the right.

6. When the trapezoid on the right is revolved once with side DC as the axis of revolution, answer the following questions.

- 1) Sketch the solid of revolution.
- 2) Find the volume of the solid.

[Various ways of looking at solids]
7. What solid is the solid shown in each projection below made up of?

- 1) \( \begin{array}{c}
\text{Cube} \quad \text{Cone} \quad \text{Cylinder}
\end{array} \)
- 2) \( \begin{array}{c}
\text{Sphere} \quad \text{Cylinder}
\end{array} \)

8. Suppose you make a solid from the cube on the right. The edges of the new solid are the segments made by linking the four vertices D, B, E, and G of the cube. When each side of the cube is 9 cm, answer the following questions.

- 1) How many faces does the new solid have?
- 2) Find the volume of the new solid.
Let's review [Organizing and making use of data]

1. The frequency distribution table below shows the chest circumference of 20 boys in a class. Answer the following questions.

<table>
<thead>
<tr>
<th>Class (cm)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least 65</td>
<td>2</td>
</tr>
<tr>
<td>65 ~ 70</td>
<td>4</td>
</tr>
<tr>
<td>70 ~ 75</td>
<td>9</td>
</tr>
<tr>
<td>75 ~ 80</td>
<td>3</td>
</tr>
<tr>
<td>80 ~ 85</td>
<td>2</td>
</tr>
<tr>
<td>85 ~ 90</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
</tr>
</tbody>
</table>

- (1) Which class has the highest frequency?
- (2) Find the relative frequency of the 80~85 cm class.

2. The histogram below shows the weight of students in a class. Answer the following questions.

- (1) How many students are there in this class?
- (2) Find the class value of the class to which the 12th heaviest student belongs.
- (3) Find the relative frequency of the 50~55 kg class by rounding it to two decimal places.

3. The table below shows the distance 20 students travel from their houses to the school. Answer the following questions about the calculation of their mean travel distance.

<table>
<thead>
<tr>
<th>Class (km)</th>
<th>Class value (km)</th>
<th>Frequency</th>
<th>(Class value x Frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>at least 0</td>
<td>0 ~ 0.6</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>0.6 ~ 1.2</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>1.2 ~ 1.8</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1.8 ~ 2.4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2.4 ~ 3.0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

- (1) Fill in the blanks with suitable numbers.
- (2) Find their mean travel distance.

4. The table below shows the quiz scores of students in a class. Answer the following questions.

<table>
<thead>
<tr>
<th>Score</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>45</td>
</tr>
</tbody>
</table>

- (1) Find the median.
- (2) Find the mode.

5. Answer the following questions.

- (1) Express 7100 km as an approximate value with a two-digit significant figure.
- (2) Express 483000 mg as an approximate value with a three-digit significant figure.
Final test

1. Calculate.
   (1) \(7 + \frac{5}{6} \times (-12)\)
   (2) \(\frac{1}{9} - 5 \div \left(-\frac{2}{3}\right)^2\)
   (3) \(0.8x - 1.5 - (1.4x - 3)\)
   (4) \(3(4x + 5) - 6(-3x + 2)\)
   (5) \(6\left(\frac{1}{3}x - 3\right) + \left(\frac{1}{4}x - 1\right) \times (-12)\)
   (6) \(\frac{2(a - 1)}{3} - \frac{3(2a - 1)}{2}\)

2. Answer the following questions.
   (1) When \(a = -4\), find the value for \(2a^2 - 3a\).

   (2) When \(-1 < a < 0\), which is the second greatest number, \(a\), \(-a\), \(a^2\), or \(-a^2\)?

3. Solve the following equations.
   (1) \(3(x - 3) + 5 = 2(3x - 5)\)
   (2) \(0.2(0.8x - 4) = 0.6(0.1x + 0.7)\)

4. There were five kinds of products A, B, C, D, and E. Their prices were 100 yen, 120 yen, 140 yen, 160 yen, and 200 yen apiece, respectively. You bought the same number of A, B, and C, and the same number of D and E. The total number of pieces you bought was 24, and you paid 3600 yen. Answer the following questions.
   (1) When you bought \(x\) pieces of product A, express how many pieces of D you bought in terms of \(x\).

   (2) Find how many pieces of A you bought.

---

Score: 0/100

1. Calculate.
   (1) \(7 + \frac{5}{6} \times (-12)\)  
   (2) \(\frac{1}{9} - 5 \div \left(-\frac{2}{3}\right)^2\)  
   (3) \(0.8x - 1.5 - (1.4x - 3)\)  
   (4) \(3(4x + 5) - 6(-3x + 2)\)  
   (5) \(6\left(\frac{1}{3}x - 3\right) + \left(\frac{1}{4}x - 1\right) \times (-12)\)  
   (6) \(\frac{2(a - 1)}{3} - \frac{3(2a - 1)}{2}\)

2. Answer the following questions.
   (1) When \(a = -4\), find the value for \(2a^2 - 3a\).

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   (1) When you bought \(x\) pieces of product A, express how many pieces of D you bought in terms of \(x\).

   (2) Find how many pieces of A you bought.
5. In the figure on the right, line $\ell$ is a graph of proportion and line $m$ is a graph of inverse proportion. Both lines pass through $A(4, 6)$. Point $P$ is on line $\ell$ and its $x$-coordinate is 8. Point $Q$ is the point where line $m$ intersects the line that passes through $P$ and is parallel to the $y$-axis. Answer the following questions.

(1) Write the expressions to indicate lines $\ell$ and $m$.

(2) Find the length of segment $PQ$.

(3) Find the area of $\triangle AOQ$.

6. Answer the following questions. Use $\pi$ for the ratio of the circumference of a circle to its diameter.

(1) Line $\ell$, point $A$, and point $B$ are given in the figure on the right. Point $A$ is on line $\ell$, and point $B$ is not on line $\ell$. Construct point $P$ so that it is at the same distance from point $A$, point $B$, and line $\ell$.

(2) Hemisphere $O$ with radius 6 cm is given in the figure on the right. When rotating it $30^\circ$ with $B$, which is one end of diameter $AB$, as the center of rotation, answer the following questions.

$\text{(1)}$ Find the circumference of the shaded part.

$\text{(2)}$ Find the area of the shaded part.

7. Answer the following questions about the cube on the right.

(1) Find the face that is parallel to the segment linking points $B$ and $D$.

(2) What kind of triangle is the one with vertices $B$, $D$, and $E$?

(3) When you make a pyramid with vertices $B$, $E$, $F$, and $G$, how many times as great is its volume as the volume of this cube?
Signed numbers
◆ Basic calculation with signed numbers◆
1. Sum of two numbers with the same sign
   \(-2 + (-5) = -(2 + 5) = -7\)
2. Sum of two numbers with different signs
   \((-3) + (-7) = -(7 - 3) = -4\)
3. Subtraction \(\rightarrow\) Add the same number with a different sign
   \((-4) - (-9) = (-4) + (+9) = +5\)
4. Product or quotient of two numbers with the same sign
   \(\rightarrow\) Product or quotient of their absolute values with the positive sign
5. Product or quotient of two numbers with different signs
   \(\rightarrow\) Product or quotient of their absolute values with the negative sign

Algebraic expressions
◆ How to express products and quotients◆
1. \(y \times 8 \times x = 8xy\)
   \(\Rightarrow\) Remove \(\times\) symbols. Write numbers in front of letters.
2. \(a \times x \times a = a^2\)
   \(\Rightarrow\) The product of a letter and itself is written using exponents of powers.
3. \((x - y) ÷ z = \frac{x - y}{z}\)
   \(\Rightarrow\) Write in fraction form without \(÷\) symbols.

◆ Calculating linear expressions◆
1. \(2x + 3 + 5x - 8 = (2 + 5)x + (3 - 8) = 7x - 5\)
2. \(3(2x + 7) = 3 \times 2x + 3 \times 7 = 6x + 21\)
3. \((15a - 9) ÷ 3 = \frac{15a}{3} - \frac{9}{3} = 5a - 3\)

Equations
◆ How to solve equations◆
1. \(6x + 10 = 3x - 2\)
   \(\Rightarrow\) Transpose terms
2. \(6x - 3x = -2 - 10\)
   \(\Rightarrow\) Simplify
3. \(3x = -12\)
   \(\Rightarrow\) Divide both sides by the coefficient of \(x\)
4. \(x = -4\)

◆ Property of proportional expressions◆
If \(a : b = c : d, ad = bc\).

◆ How to solve word problems of equations◆
1. Let \(x\) be an unknown quantity.
2. Find an equivalent relationship between two quantities and write an equation.
3. Solve the equation to find the solution.

Proportion and inverse proportion
◆ Proportion \(y = ax\)◆
1. \(y\) is proportional to \(x \Leftrightarrow y = ax\)
2. If \(x \neq 0, \frac{y}{x} = a\) (Constant)
◆ Inverse proportion \(y = \frac{a}{x}\)◆
1. \(y\) is inversely proportional to \(x \Leftrightarrow y = \frac{a}{x}\)
2. \(xy = a\) (Constant)
◆ Graph of proportion \(y = ax\)◆
Line that passes through the origin
- Rise up to the right
- Fall down to the right
◆ Graph of inverse proportion \(y = \frac{a}{x}\)◆
Two curves symmetric about the origin called hyperbolas

Space figures
◆ Sectors◆
1. \(\ell = 2\pi r \times \frac{a}{360}\), \(S = \pi r^2 \times \frac{a}{360}\)
2. \(S = \frac{1}{2} \ell r\)

Planefigures
◆ Arcs and chords, Tangents◆
- Arc: Part of the circumference of a circle
- Chord: Segment linking two points on the circumference of a circle
- A tangent is perpendicular to the radius passing through the tangent point.

◆ Figure Transformation◆
- Translation: Slide a figure in a single direction for a certain distance.
- Rotation: Turn a figure at a certain angle around a central point.
- Reflection: Turn a figure over a single central line.

◆ Figure construction◆
- Perpendicular bisector
- Angle bisector

Organizing and making use of data
◆ Organizing data◆
- Frequency distribution table
- Histogram

<table>
<thead>
<tr>
<th>Class (m)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ~ 6</td>
<td>3</td>
</tr>
<tr>
<td>6 ~ 7</td>
<td>8</td>
</tr>
<tr>
<td>7 ~ 8</td>
<td>6</td>
</tr>
<tr>
<td>8 ~ 9</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>19</td>
</tr>
</tbody>
</table>

- (Relative frequency) \(=\) \(\frac{\text{Frequency of each class}}{\text{Total frequency}}\)

◆ Representative values◆
- How to calculate the mean using a frequency distribution table
- Sum of (Class value) \times (Frequency) \(=\) \(\text{Total frequency}\)
- Median: Middle value among the values of a data set when they are arranged in size order. (Even number of data items \(\Rightarrow\) Mean of the two middle values)
- Mode: Value that appears most frequently in a set of data

Significant figures◆
3400 g expressed as a three-digit significant figure is \(3.40 \times 10^3\) g.